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# A THEORETICAL REAL-GAS ANALYSIS OF THE EXPANSION TUNNEL

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Glenn D. Norfleet and F. C. Loper  
ARO, Inc.

June 1966

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OF THE EXPANSION TUNNEL

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## FOREWORD

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This technical report has been reviewed and is approved.

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**ABSTRACT**

A theoretical real-gas analysis of the expansion tunnel is presented. A digital computer program, developed for this investigation, is discussed, and Fortran listings and flow charts are included. Tunnel performance, test gas slug length, and "working" parameters are given for several expansion area ratios. Driver temperature and energy requirements are given for specific cases.

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### NOMENCLATURE

|                          |   |
|--------------------------|---|
| A                        | Cross-sectional area of tube                                |
| a                        | Acoustic speed  |
| $c_p$                    | Specific heat at constant pressure                          |
| d                        | Tube inside diameter  |
| e                        | Driver energy parameter, $\Delta E/A_{6A} \Delta t_r$       |
| h                        | Enthalpy  |
| $\ell$                   | Length  |
| $\ell_1, \ell_4, \ell_8$ | Tube lengths (see Fig. 1)                                   |
| $\ell_t$                 | Test gas slug length at time of secondary diaphragm rupture |
| M                        | Mach number   |
| $M_S$                    | Shock Mach number   |
| p                        | Pressure  |
| $p_i$                    | Arc driver gas pressure prior to arc discharge              |

|              |   |
|--------------|---|
| $R$          | Gas constant  |
| $S$          | Entropy   |
| $T$          | Temperature   |
| $T_i$        | Arc driver gas temperature prior to arc discharge                         |
| $U$          | Velocity  |
| $U_S$        | Shock velocity  |
| $x$          | Axial distance along tube   |
| $Z$          | Compressibility factor  |
| $\gamma$     | Ratio of specific heats   |
| $\Delta E$   | Driver power supply stored energy   |
| $\Delta t_r$ | Ideal run time  |
| $\Delta t_w$ | Wave transit time (see Fig. 1)  |
| $\eta$       | Arc driver efficiency, effective driver energy addition/<br>stored energy |
| $\xi$        | Driver power factor, $\Delta E / \ell_1 A_1$                              |
| $\rho$       | Density   |

**SUBSCRIPTS**

|               |  |
|---------------|--|
| 1, 2, 3, etc. | Denote various flow regions (see Fig. 1) |
| a             | Standard conditions                      |
| (max)         | Maximum                                  |
| o             | Stagnation value                         |
| (opt)         | Optimum                                  |
| r             | Reference condition                      |

**DUAL SUBSCRIPTS**

Example:  $p_{ij} = p_i / p_j$



## SECTION I INTRODUCTION

An aerodynamic test device utilizing an unsteady expansion to accelerate shock heated gas was first proposed by Resler and Bloxsom (Ref. 1) and was treated briefly by Hertzberg et al. (Ref. 2). Trimpi (Ref. 3) made a detailed theoretical study of the expansion tube, a device in which the entire expansion from the shock heated condition to the test condition is performed unsteadily. In both Refs. 2 and 3 it was suggested that an area change could be added downstream of the unsteady expansion so that part of the expansion would be performed steadily. Trimpi and Callis (Ref. 4) later did a perfect gas analysis of such a device, which is called an expansion tunnel. The basic wave diagrams for both the expansion tube and tunnel are shown in Fig. 1.

For a given density level the test gas velocity obtainable with an expansion tube is approximately twice that obtained by a shock tunnel with the same driver (Ref. 3), as shown in Fig. 2. In spite of this considerable performance gain, little effort has been made to develop an operable expansion tube. To a large degree this hesitancy comes from the expected problems associated with the device.

One of the more questionable aspects of the expansion tube is the uniformity of the test gas. Some insight into the problem can be gained from Fig. 3, which gives the ratio of the test gas slug length after shock heating to the tube diameter. This figure shows that the test gas is in close proximity to the secondary diaphragm at the time of rupture. Although the secondary diaphragm can be very thin, it does require a finite time to rupture. There is, then, a shock wave reflected from the diaphragm which very quickly weakens to the vanishing point. Since the diaphragm will be bulged, this reflection will not be planar. At least the initial portion, and possibly all, of the test gas passes through this shock wave which is not uniform in either space or time.

For the perfect gas case the addition of the steady expansion, as in the expansion tunnel, is very effective in alleviating the test gas slug length problem.<sup>1</sup> The steady expansion is detrimental, however, in terms of test gas velocity obtainable with a given driver.<sup>2</sup> The optimum expansion tunnel design will probably require some compromise between maximum test gas slug length and maximum velocity.

---

<sup>1</sup>See Fig. 22 of Ref. 4.

<sup>2</sup>See Fig. 5 of Ref. 4.

The investigation reported herein was undertaken in order to determine significant expansion tunnel performance parameters based upon a real air<sup>3</sup> test gas and to generate the proper working charts for future expansion tunnel design and operation (Appendix I).

## SECTION II CALCULATIONS

### 2.1 EXPANSION TUNNEL CALCULATIONS

This investigation is concerned with the theoretical possibilities of an expansion tunnel as a high velocity flight duplication<sup>4</sup> device. It is meaningful, then, to determine the significant parameters such as shock strengths, pressures, and tube lengths in terms of duplication altitude and flight velocity. The bulk of these computations was done with a digital computer program which is briefly described below. A detailed description including the Fortran listing and flow diagram is given in Appendixes II and III. The test gas slug length,  $l_t$ , and accelerating tube charge pressure were calculated by hand as described in Sections 2.1.2 and 2.1.3, respectively.

#### 2.1.1 The Expansion Tunnel Program

The expansion tunnel program is designed to determine flow parameters and tube lengths of interest for given altitude, velocity, and expansion area ratio. The input and output data for this program are shown in Table I. The expansion tunnel portion of Fig. 1 shows the various flow regions.

The general procedure is as outlined below.

1. For a given flight altitude, free-stream pressure and temperature are obtained by table look-up using the data of Ref. 5. Enthalpy,  $h_{sA}$ , and entropy,  $S_{sA}$ , are calculated using the perfect gas equations:

$$h_{sA} = c_p T_{sA}$$

$$S_{sA} - S_r = c_p \ln \frac{T_{sA}/T_r}{\left(\frac{P_{sA}}{P_r}\right)^{\frac{\gamma-1}{\gamma}}}$$

---

<sup>3</sup>Thermodynamic and chemical equilibrium was assumed for all calculations included herein.

<sup>4</sup>Duplication herein refers to the complete matching of ambient properties and ambient chemistry together with the required flow velocity.

The perfect-gas equations are valid for altitudes of interest.

2. The continuity and energy equations for the steady isentropic expansion from 6 to 6A are combined, yielding

$$\rho_6 A_6 (2h_{o_{6A}} - 2h_6)^{1/2} = \rho_{6A} U_{6A} A_{6A}$$

This equation and the equation of state, represented by the Table of Thermodynamic Properties (Refs. 6 and 7) are solved (for S constant) simultaneously for  $\rho_6$  and  $h_6$ .

3. The unsteady expansion from 2 to 6 and the shock crossing 1 to 2 must be performed simultaneously since the limit of the expansion is determined by the shock crossing.

For the unsteady isentropic expansion,

$$U_6 - U_2 = - \int_{h_2}^{h_6} \left( \frac{dh}{a} \right)_S$$

From the tables of thermodynamic properties (Refs. 5 and 6) through the unsteady expansion:

$$a = f_1(h, S)$$

at 2:

$$p_2 = f_2(h_2, S_2)$$

$$\rho_2 = f_3(h_2, S_2)$$

Across the shock,  $M_{S_1}$ :

$$p_2 + \rho_2 (U_{S_1} - U_2)^2 = p_1 + \rho_1 U_{S_1}^2 \quad (\text{momentum equation})$$

$$h_2 + 1/2 (U_{S_1} - U_2)^2 = h_1 + 1/2 U_{S_1}^2 \quad (\text{energy equation})$$

$$\rho_2 (U_{S_1} - U_2) = \rho_1 U_{S_1} \quad (\text{continuity equation})$$

For the gas in region 1:

$$p_1 = \rho_1 R_1 T_1 \quad (\text{ideal gas equation of state})$$

The charge gas temperature,  $T_1$ ,<sup>5</sup> and enthalpy,  $h_1$ , are inputs to the program. The above equations are solved for the required flow parameters in regions 1 and 2, assuming the unsteady expansion to be isentropic.

---

<sup>5</sup>For the calculations reported herein,  $T_1 = 296^\circ\text{K}$ .

The accelerating tube length is calculated from

$$\ell_8 / \Delta t_r = U_6 (M_6 - 1)$$

The tube length ratio  $\ell_1 / \ell_8$  is optimum<sup>6</sup> when the (U + a) wave reflected from the 2 - 3 interface overtakes the tail of the expansion at the test section as shown in Fig. 1. The time required for the passage of this reflected wave,  $\Delta t_w / \ell_8$ , is calculated by integration through the unsteady expansion. The technique used to determine  $\ell_1 / \ell_8$  is described in Appendices II and III and is similar to that of Ref. 8.

The theoretical model of this program was based upon the following simplifying assumptions:

1. Air in regions 1 and 6A is assumed to be ideal.
2. Air in regions 2 and 6 is assumed to be in thermodynamic equilibrium.
3. Flow is inviscid and one dimensional throughout.
4. Diaphragm rupture is instantaneous with no losses.
5. The expansion nozzle has no length and therefore zero "start" time.

### 2.1.2 Test Gas Slug Length

The test gas slug length,  $\ell_t$ , was calculated from the appropriate form of the continuity equation,

$$\rho_2 \ell_t A_2 = \rho_{6A} U_{6A} \Delta t_r A_{6A}$$

### 2.1.3 Accelerating Tube Charge Pressure

The accelerating tube charge pressure,  $p_8$ , was determined for a given  $a_8$  using  $U_6$  and  $p_6$  from:

$$p_8 = p_7 (p_8/p_7) = p_6 (p_8/p_7)$$

and

$$p_6 = p_7 \quad (\text{across interface})$$

The ratio of  $p_8/p_7$  was obtained from Ref. 9 for a given

$$U_6/a_8 = U_7/a_8 \quad (\text{across interface}).$$

---

<sup>6</sup>i.e., minimum  $\ell_1$  for maximum run time with a given  $\ell_8$ .

## 2.2 DRIVER CALCULATIONS

The loss in performance associated with the steady expansion can, within limits, be offset by using a higher performance driver. While driver design, per se, is not within the scope of this investigation, a knowledge of driver requirements becomes important in assessing the significance of the performance loss.

Of particular interest are driver pressure, temperature, and energy requirements. Driver pressure and temperature were calculated using an existing shock tube program, but driver energy (and driver optimization based upon energy requirements) was determined by hand calculations.

### 2.2.1 The Shock Tube Program

The shock tube program used in this study determines flow conditions for given charge conditions in a two- or three-stage shock tube. The program was used to determine driver charge conditions for a given  $M_{S1}$  and  $p_1$ . This solution is obtained in the usual manner of expanding the driver gas to match pressure and velocity at the 2 - 3 interface. The snock crossing is performed much as is done in the expansion tunnel program except that the thermodynamic properties are obtained by an empirical surface fit (Ref. 10) to the data of Ref. 11. The basic thermodynamic data used in the two machine programs differ in that the data for the expansion tunnel program include intermolecular force effects whereas the data for the shock tube program do not. The shock tube program was used only to determine shock strength,  $M_{S1}$ , for given driver and driven tube conditions. Experience has shown that values of  $M_{S1}$  calculated with either set of data are in good agreement. The inconsistency in thermodynamic properties applies only to calculations involving the driver, and there its effects are felt to be insignificant.

This program will accept driver/driven area ratios of any value; however, for  $A_{41} < 1$  and low values of  $p_{41}$ , upstream-facing secondary shocks are possible. If such a shock is standing in the area change, as opposed to moving downstream, the program will not give a solution. In such cases the solution was obtained by hand calculations.

Program calculations are based upon the following simplifying assumptions:

1. Gases are assumed to be perfect except for the shock heated region 2 which is taken as real air in thermodynamic equilibrium.

2. Flow is inviscid and one dimensional throughout.
3. Diaphragm rupture is instantaneous with no losses.

### 2.2.2 Driver Energy and Optimization

Driver energy, which becomes highly significant in the case of arc heating since its magnitude is reflected directly in the cost of a power supply, was calculated assuming:

1. Helium as the driver gas (perfect gas assumed).
2. The power supply is of the fast discharge type, with a discharge time of the order of 100  $\mu$ sec. Arc efficiency data (Fig. 4) were obtained from Refs. 12 and 13 and are applicable to a fast discharge system.

Driver energy per unit volume for a constant volume energy addition process is given by

$$\frac{\Delta E}{A_4 \ell_4} = \frac{1}{\gamma_4 - 1} \frac{p_4}{\eta} \frac{T_4 - T_i}{T_4}$$

Multiplying by the tube length and area ratios yields the energy parameter,  $e$ ,

$$e \equiv \frac{\Delta E}{A_{6A} \Delta t_r} = \frac{1}{\gamma_4 - 1} \frac{p_4}{\eta} \frac{T_4 - T_i}{T_4} \left( \frac{A_6}{A_{6A}} \right) \left( \frac{A_4}{A_1} \right) \left( \frac{\ell_4}{\ell_1} \right) \left( \frac{\ell_1}{\ell_8} \right) \left( \frac{\ell_8}{\Delta t_r} \right)$$

where  $p_4$  and  $T_4$  were given by the shock tube program

$\frac{\ell_4}{\ell_1}$  was calculated by hand using local values of velocity and acoustic velocity from the shock tube program

$\frac{\ell_8}{\Delta t_r}$  and  $\frac{\ell_1}{\ell_8}$  were given by the expansion tunnel program

$\eta$  was taken from Fig. 4

$T_i$  was assumed to be 296°K

and  $A_6 = A_1$

There are an infinite number of combinations of  $T_4$ ,  $p_4$ , and  $A_{41}$  which will yield identical theoretical driver performance. In order to give meaning to energy requirements, it is necessary to choose the combination of these variables which will yield the given performance while using minimum energy, i. e., the driver must be optimized. In order to optimize the driver, a power factor is defined as:

$$\xi = \frac{\Delta E}{A_1 \ell_1} = \frac{\Delta E}{A_4 \ell_4} \frac{A_4}{A_1} \frac{\ell_4}{\ell_1}$$

The driver is optimized when, for a given  $M_{S1}$ ,  $p_1$ , and  $T_1$ , the value of  $\xi$  (driver energy per driven tube unit volume) is minimum. The actual optimization process consisted of choosing values of  $T_4$  and  $A_{41}$ , calculating  $p_4$ , and then varying  $T_4$  and  $A_{41}$  until a minimum was reached in  $\xi$ .

### SECTION III RESULTS<sup>7</sup>

#### 3.1 PERFORMANCE

Performance calculations in terms of velocity and altitude were made for expansion area ratios of 1, 10, 100, and 1000. Required shock strength,  $M_{S1}$ , charge pressure,  $p_1$ , and compressibility factor,  $Z_2$ , are presented in Fig. 5. The parameters  $M_{S1}$  and  $p_1$  were chosen because of their wide acceptance as independent variables in shock tube work (particularly in shock-crossing calculations). Their values give a general indication of driver requirements. The value of  $Z_2$  is an indication of the level of dissociation and ionization in the shock heated gas in region 2. Bray (Ref. 14) has shown that for the case of a steady expansion in which the flow upstream of the expansion is not frozen, the mole fraction of frozen constituents after the expansion is a very weak function of the ionization-dissociation level prior to the expansion.<sup>8</sup> If the same phenomenon occurs in an unsteady expansion, then the value of  $Z_2$  is not necessarily indicative of the ionization-dissociation level in the expanded test gas. Until an analysis of recombination through an unsteady expansion is available, no meaningful comparison of test gas ionization-dissociation level for different expansion area ratios can be made.

#### 3.2 DRIVER REQUIREMENTS

In order to investigate driver requirements in more detail, driver temperature,  $T_4$ , was calculated for a constant area driver using helium at a pressure,  $p_4$ , of 5000 atm. The results are shown in Fig. 6. The drivers considered here represent rather severe requirements, but their design is believed to be within the present "state of the art". The high temperature helium drivers would probably be arc heated, although the densities are somewhat higher than normal for arc drivers.

---

<sup>7</sup>Additional results, in the form of working graphs for tunnel design and operation, are presented in Appendix I.

<sup>8</sup>Applies specifically to frozen atomic oxygen.

The driver energy optimization was done for a helium driver for the specific case of  $M_{S1} = 10$ ,  $p_1 = 10$ , and  $T_1 = 296^\circ\text{K}$ . The variation of the power factor,  $\xi$ , with driver temperature and area ratio is shown in Fig. 7. For a maximum driver pressure of 5000 atm, the optimum occurs at a temperature,  $T_4$ , of about 5000°K and an area ratio  $A_{41} \approx 1$ . The driver energy parameter,  $e$ , is shown for the optimum driver in Fig. 6. Note that these energies are for constant shock strength and charge pressure,  $p_1$ , and not constant performance. Although they cannot be used to compare energy requirements as a function of expansion area ratio, they do give single point energy requirements for an optimum driver and, therefore, some insight into power supply requirements.

In order to compare energy requirements for various expansion ratios, it is necessary to optimize the driver at the same performance level for each expansion area ratio. This gives a different value of  $M_{S1}$  and  $p_1$  for each value of  $A_{6A}/A_6$ . In order to simplify the optimization it is assumed that the optimum area ratio,  $A_{41(\text{opt})}$ , is equal to one. This assumption is reasonable for shock strengths,  $M_{S1}$ , near 10 and pressures,  $p_1$ , near 10 atm since the constant temperature curves of Fig. 7 are very flat in the region near  $A_{41} = 1$ .

Optimum driver temperature and pressure were calculated for the performance level of  $U_{6A} = 30,000$  ft/sec and altitude = 150,000 ft ( $9.3 \leq M_S \leq 12.6$ ,  $1.2 \leq p_1 \leq 10.4$  atm). Energy required for the optimum case is shown as a function of the expansion area ratio in Fig. 8. The inclusion of driver area ratio in the optimization would produce second-order changes in the curve; however, it is doubtful that this would be significant in view of the large variation of energy with expansion tunnel area ratio.

One point concerning the inviscid flow assumption seems worthy of mention here. There are two viscous effects which can greatly affect the driver energy requirements:

1. Shock attenuation (energy loss)
2. Decreased run time (mass loss).

In order to offset shock attenuation in both the driver and accelerating tubes, a more energetic driver gas will have to be used. In addition a longer driven tube, and therefore a longer driver, will be required to recover the run time lost by boundary-layer effects (see Refs. 15 and 16). In terms of driver energy requirements the two effects are additive, and it



is likely that for high  $\ell/d$  tubes the inviscid calculation will significantly underpredict driver energy requirements.

### 3.3 RUN TIME

Run time per unit length of driven and accelerating tube is presented in Fig. 9. Driver length is not included since it is not defined by altitude alone, but depends upon the particular driver conditions chosen. Usually, its length will be small compared to the combined length of the accelerating and driven tubes. It should be emphasized that the run time presented here assumes an instantaneous nozzle start. Although  $\Delta t_r/(\ell_1 + \ell_8)$  increases with increasing expansion ratios, the effect of expansion area on the actual run time will depend to a large degree on the nozzle starting process. No attempt was made in this study to determine nozzle start times; however, the "perfect start" perfect gas case is treated in Ref. 4.

### 3.4 TEST GAS SLUG LENGTH

Figure 10 gives the length of the test gas slug at the time of secondary diaphragm rupture. The parameter on the right,  $\ell_t/d_1$ , is the ratio of the test gas slug length to the diaphragm diameter for an accelerating tube length-to-diameter ratio of 200. The maximum value of  $\ell_8/d_8$ , and therefore  $\ell_t/d_1$ , is determined by viscous effects and is unknown. However, from shock tube experience, an  $\ell_8/d_8$  of 200 is quite large. Even for the large  $\ell/d$  and an expansion ratio of 1000, the test gas slug length is only one-tenth of the diaphragm diameter for high velocities.

## SECTION IV CONCLUDING REMARKS

In order to better illustrate the effect of the steady expansion, summary plots (Figs. 11 and 12) were made for a specific case. For comparison purposes the perfect gas results of Ref. 4 are also shown in Fig. 12.

Performance loss by the addition of the area change for an altitude of 150,000 ft is illustrated in Fig. 11. Since the performance lines are for a constant altitude, and therefore a constant entropy, it follows that, for a given  $M_{S1}$ ,  $p_1$  is constant. A given driver, then, would operate along a line of constant  $M_{S1}$ .

As shown in Fig. 11, the loss in velocity for a given driver can be considerable; however, it becomes significant only when the driver is of limited potential. Figure 12a illustrates temperature requirements for a specific performance ( $U_{6A} = 30,000$  ft/sec at a duplicated altitude of 150,000 ft) and a driver pressure of 5000 atm. For this performance point the large expansion area ratios would entail severe driver temperature requirements for drivers which heat the gas by heat transfer from surrounding surfaces.

Drivers which heat the gas directly, such as electric arc heated drivers, have temperature limits which are quite high and thus would not impose fundamental limitations for the performance shown here. Drivers operating in this mode are generally limited more by energy requirements since they add energy very rapidly and normally require an energy storage system which has a high relative cost. For a given test section size and run time, driver energy and, therefore, power supply costs decrease with increasing expansion area ratio (Fig. 12b).

For a given accelerating tube length-to-diameter ratio,  $\ell_g/d_g$ , large gains can be made in test slug length,  $\ell_t/d_1$  (and hopefully flow uniformity), by using large expansion area ratios (Fig. 12c). It should be noted that the parameter actually of interest is  $\ell_t/d_1(\max)$  which occurs at  $\ell_g/d_g = \ell_g/d_g(\max)$ , and is dependent upon boundary-layer growth. No attempt is made here to include boundary-layer effects; however, perfect gas calculations for a simplified model are included in Ref. 4.

In summary, an increasing expansion area ratio causes:

1. A loss in performance if the driver is limited in temperature and pressure.
2. A gain in performance if the driver is limited only in energy.
3. An increase in the test gas slug length parameter,  $\ell_t/\ell_g$ .

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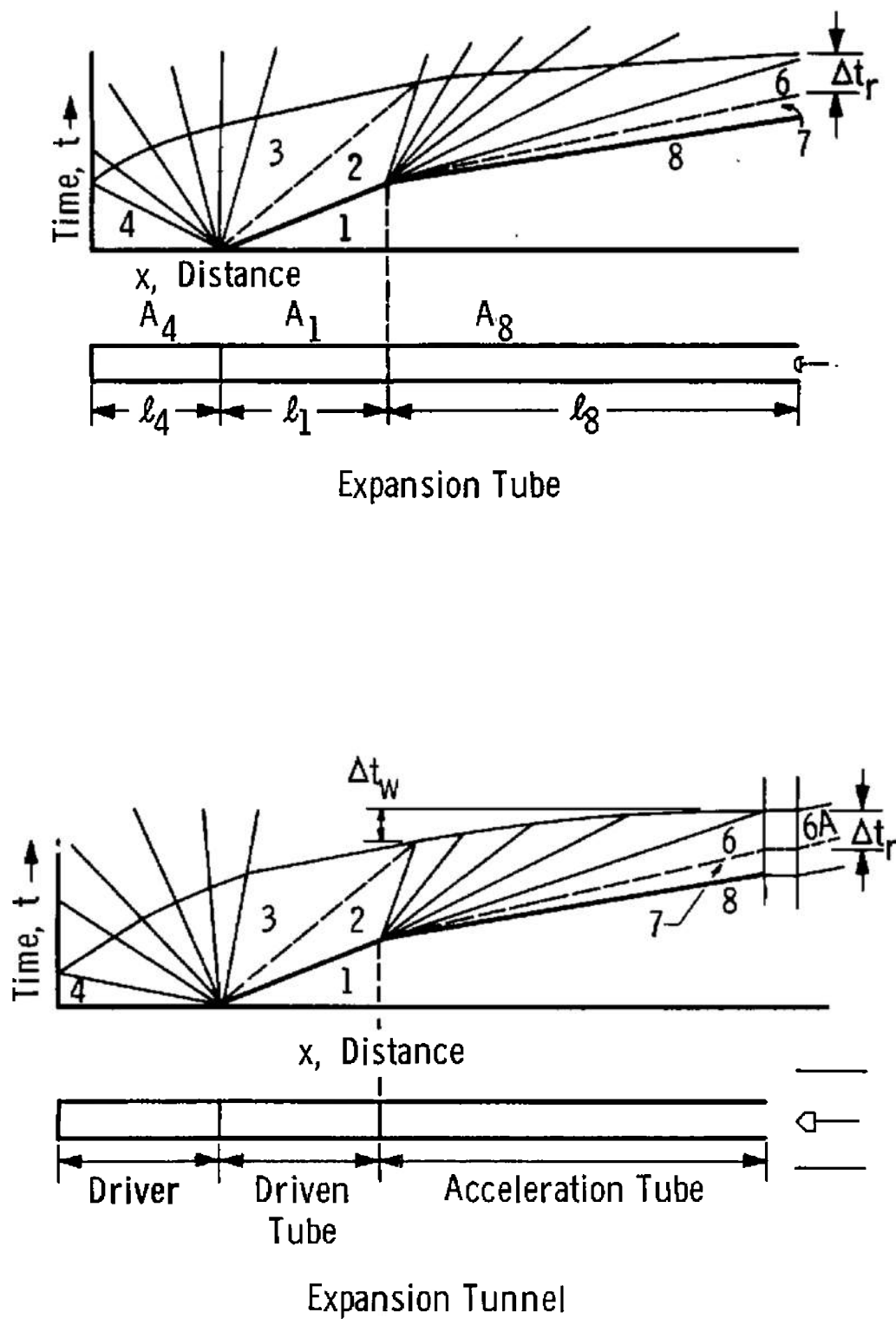


Fig. 1 Wave Diagrams – Expansion Tube and Expansion Tunnel

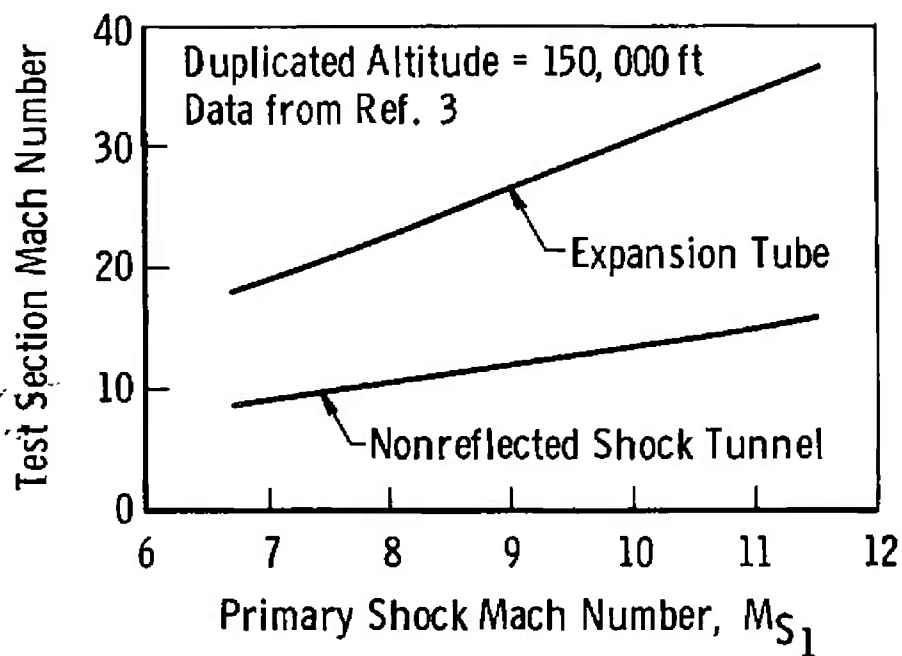


Fig. 2 Comparison of Expansion Tube and Shock Tunnel

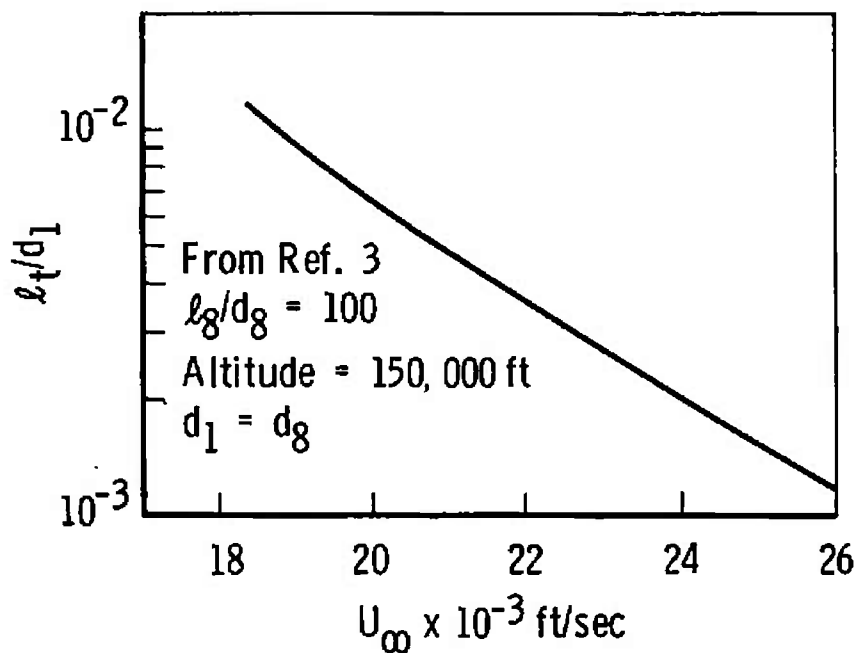


Fig. 3 Test Gas Slug Length - Expansion Tube

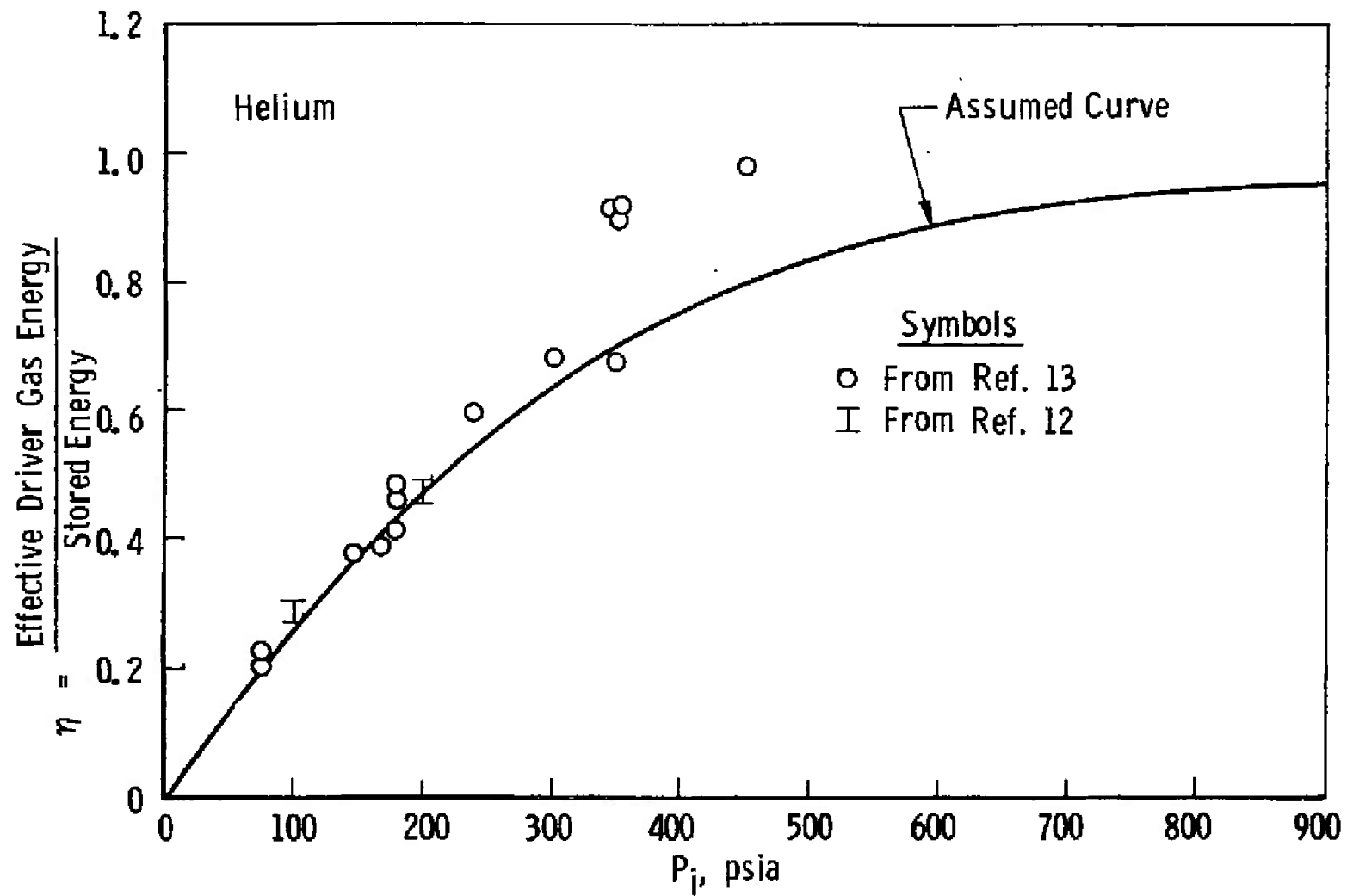
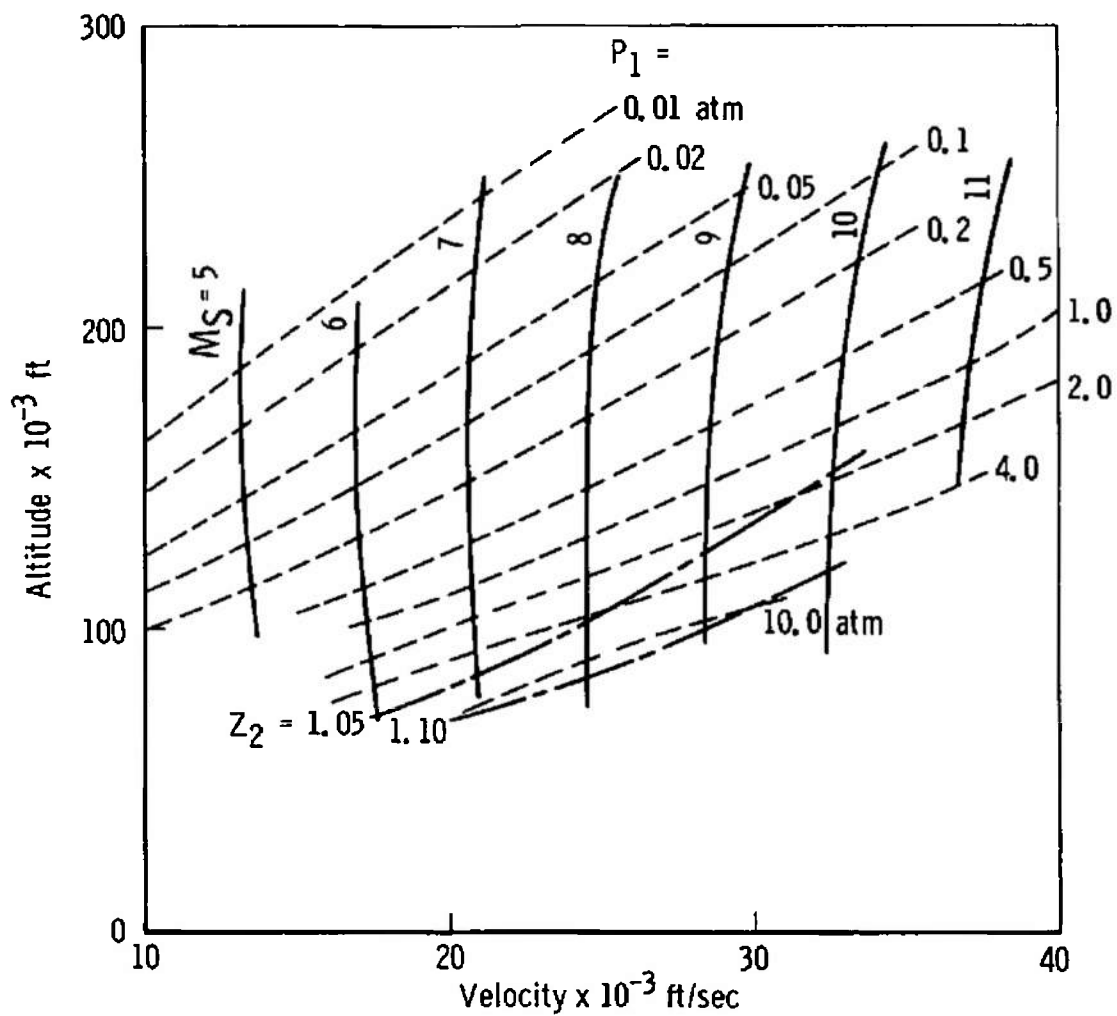


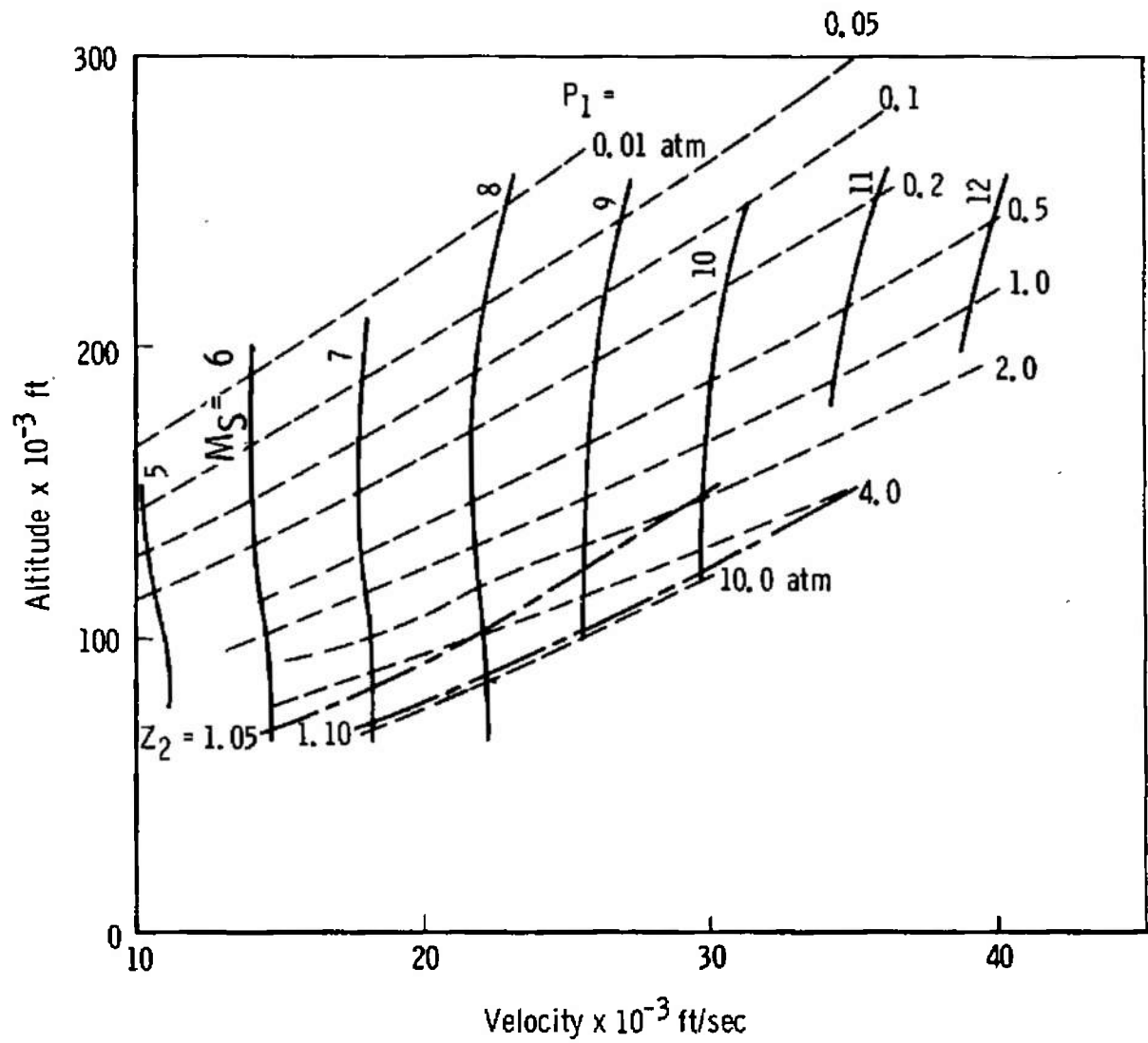
Fig. 4 Arc Driver Efficiency



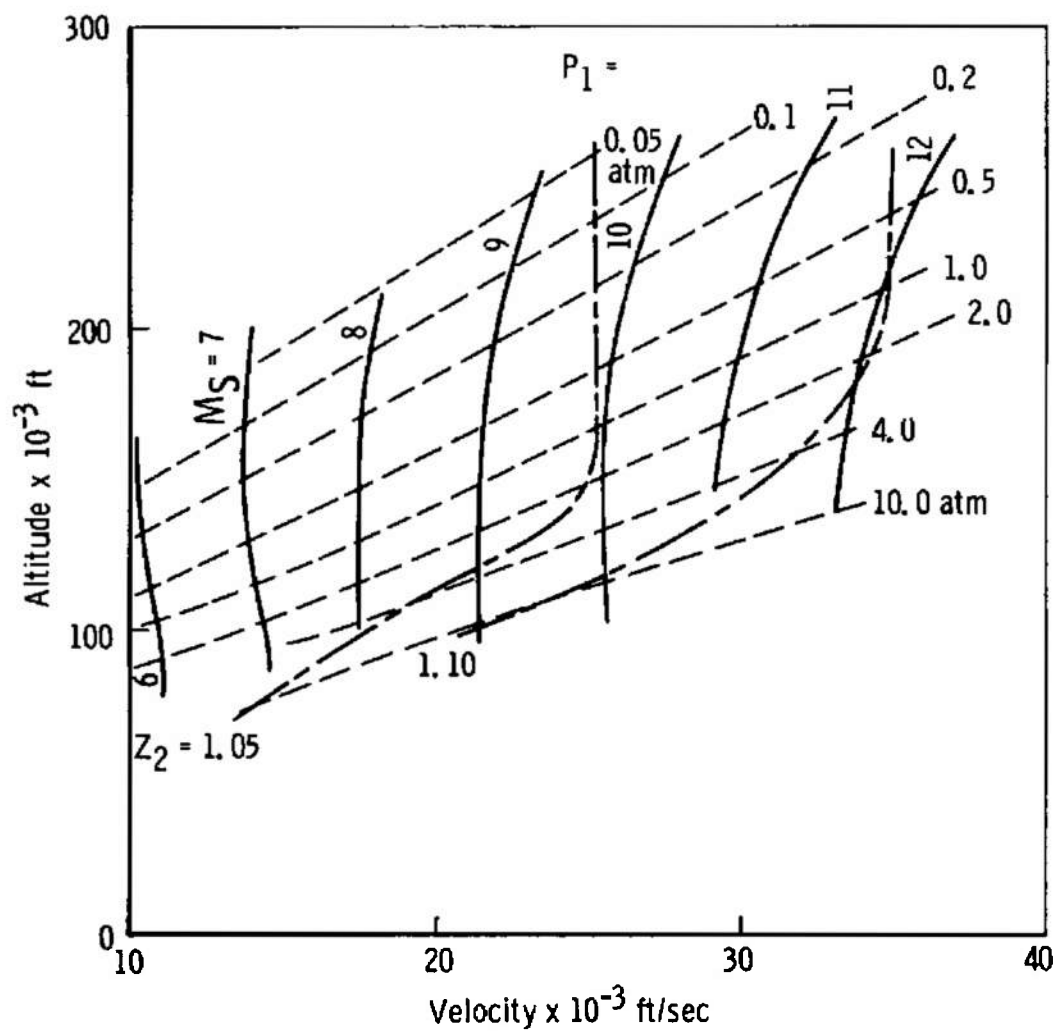
a.  $A_{6A}/A_6 = 1$

**Fig. 5 Performance Map**

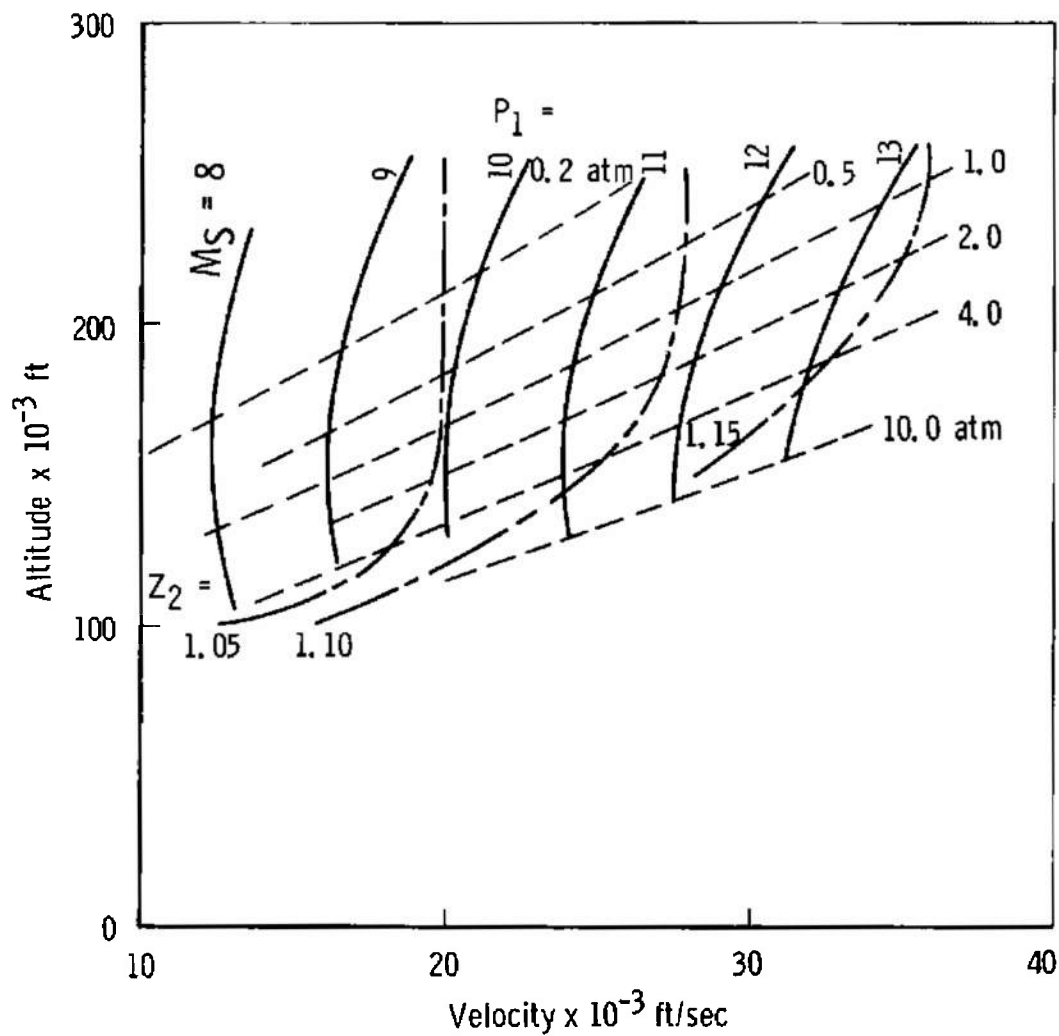




b.  $A_{6A}/A_6 = 10$   
 Fig. 5 Continued

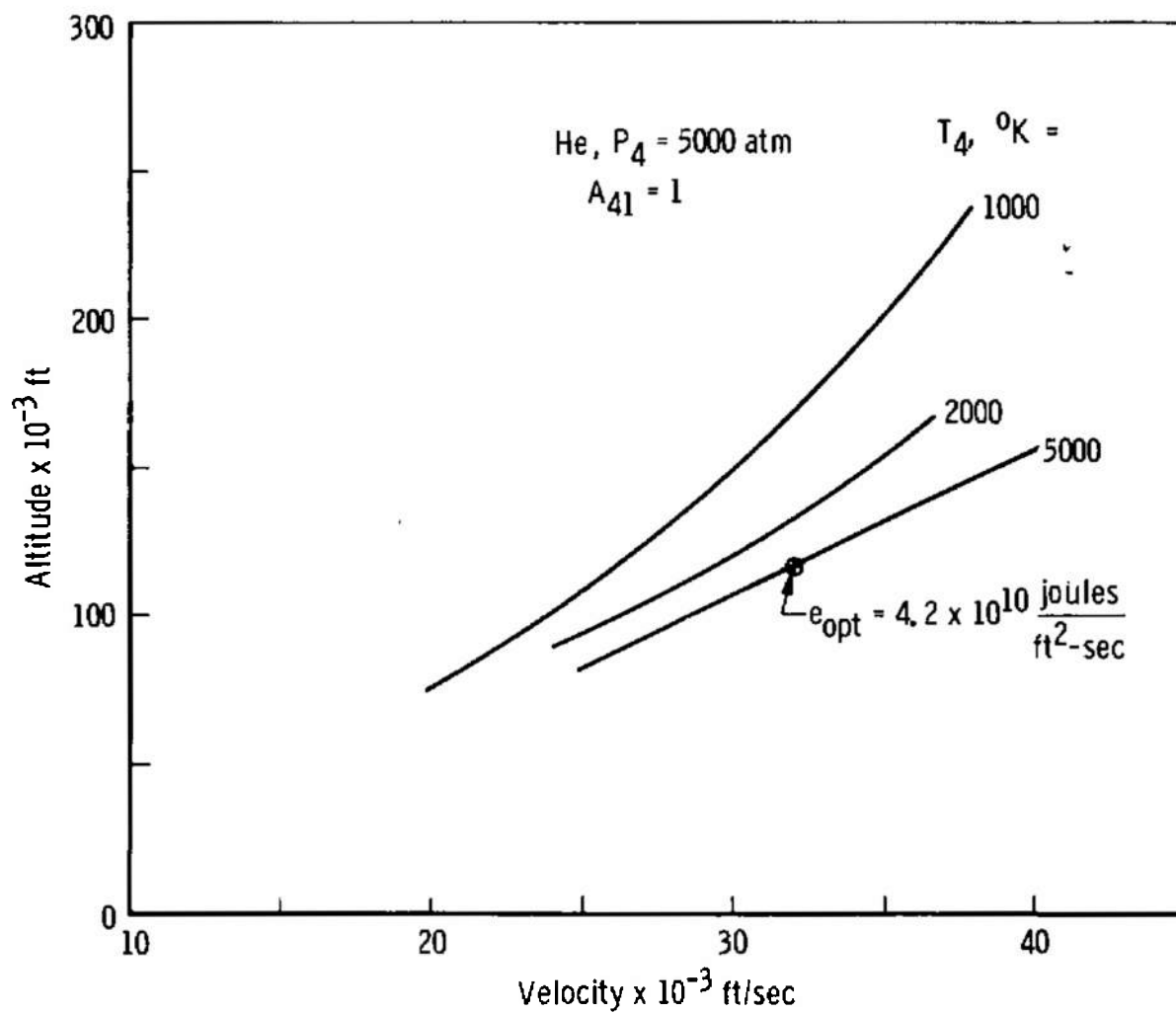


c.  $A_{6A}/A_6 = 100$   
 Fig. 5 Continued



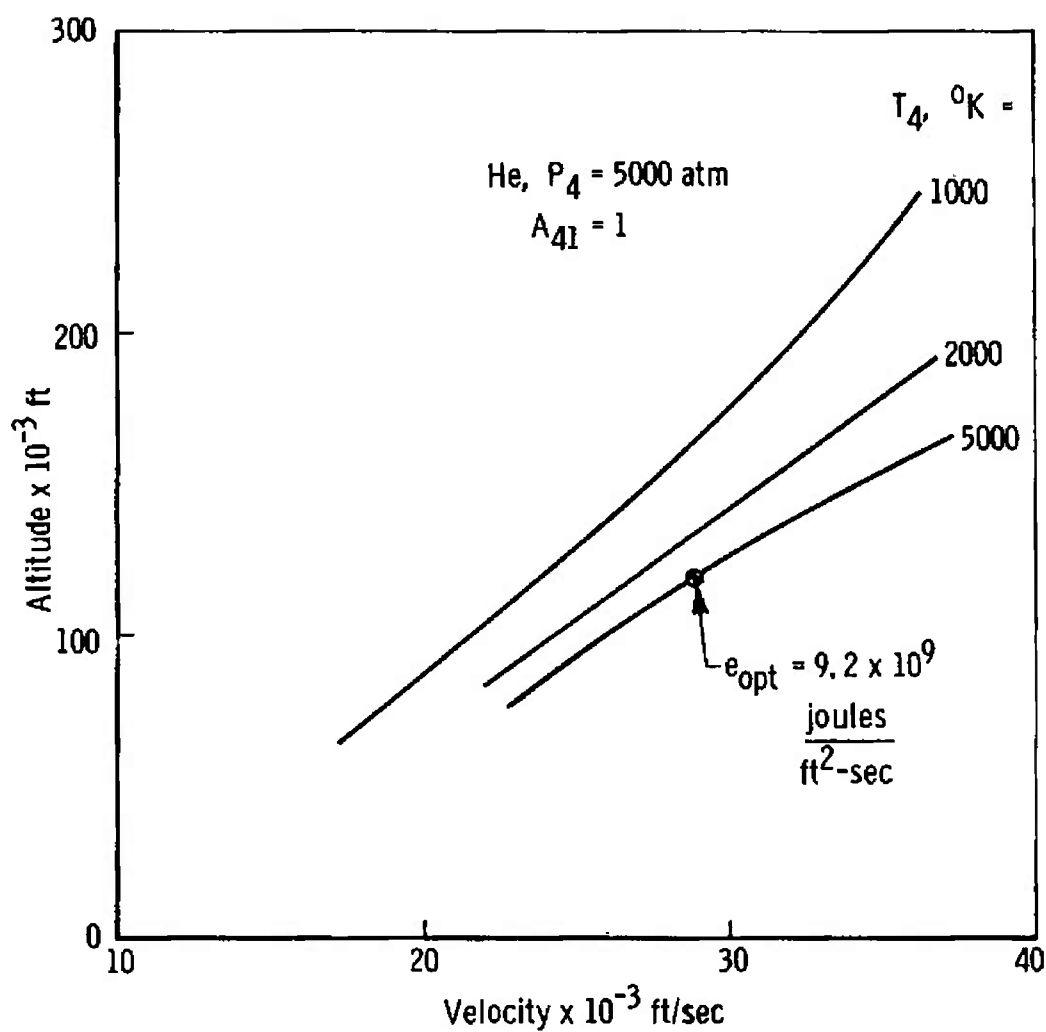
d.  $A_{6A}/A_6 = 1000$

Fig. 5 Concluded

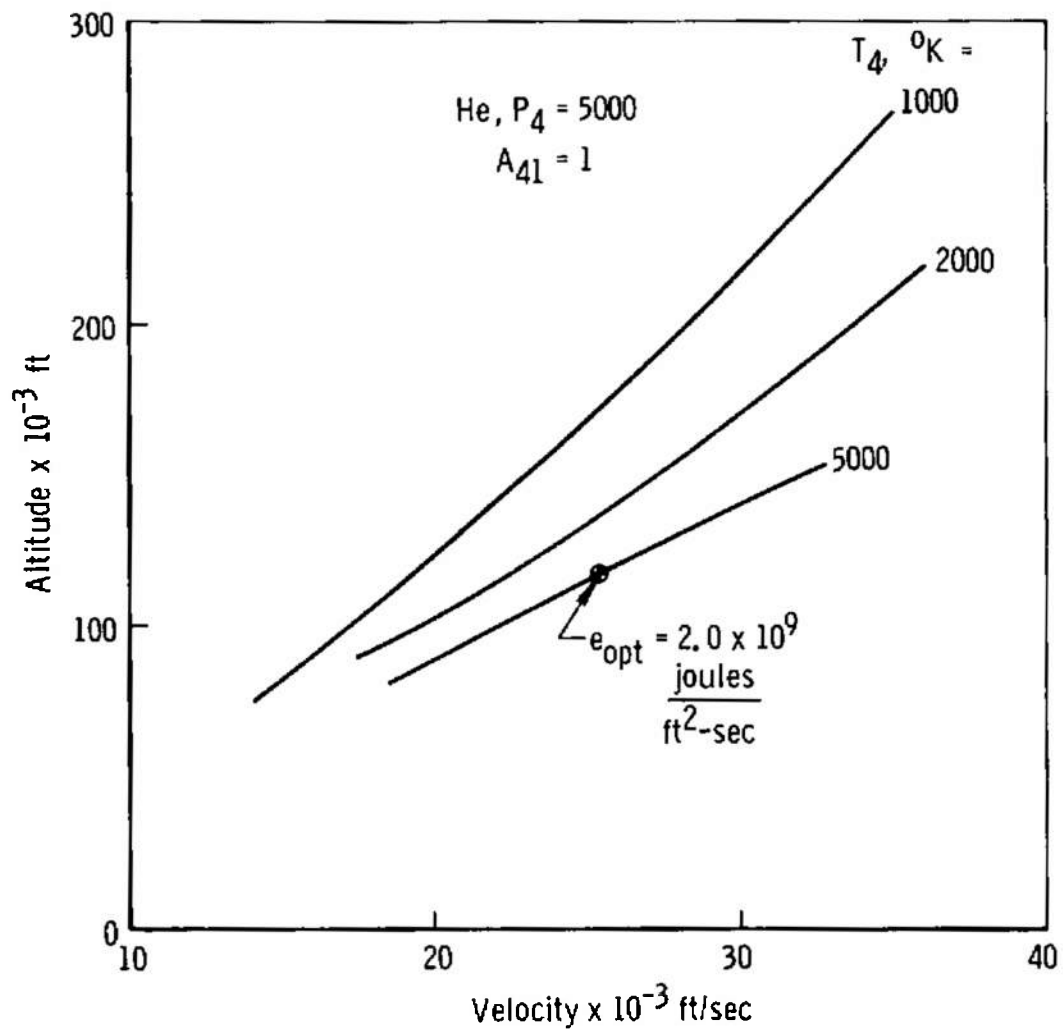


$$a. A_{6A}/A_6 = 1$$

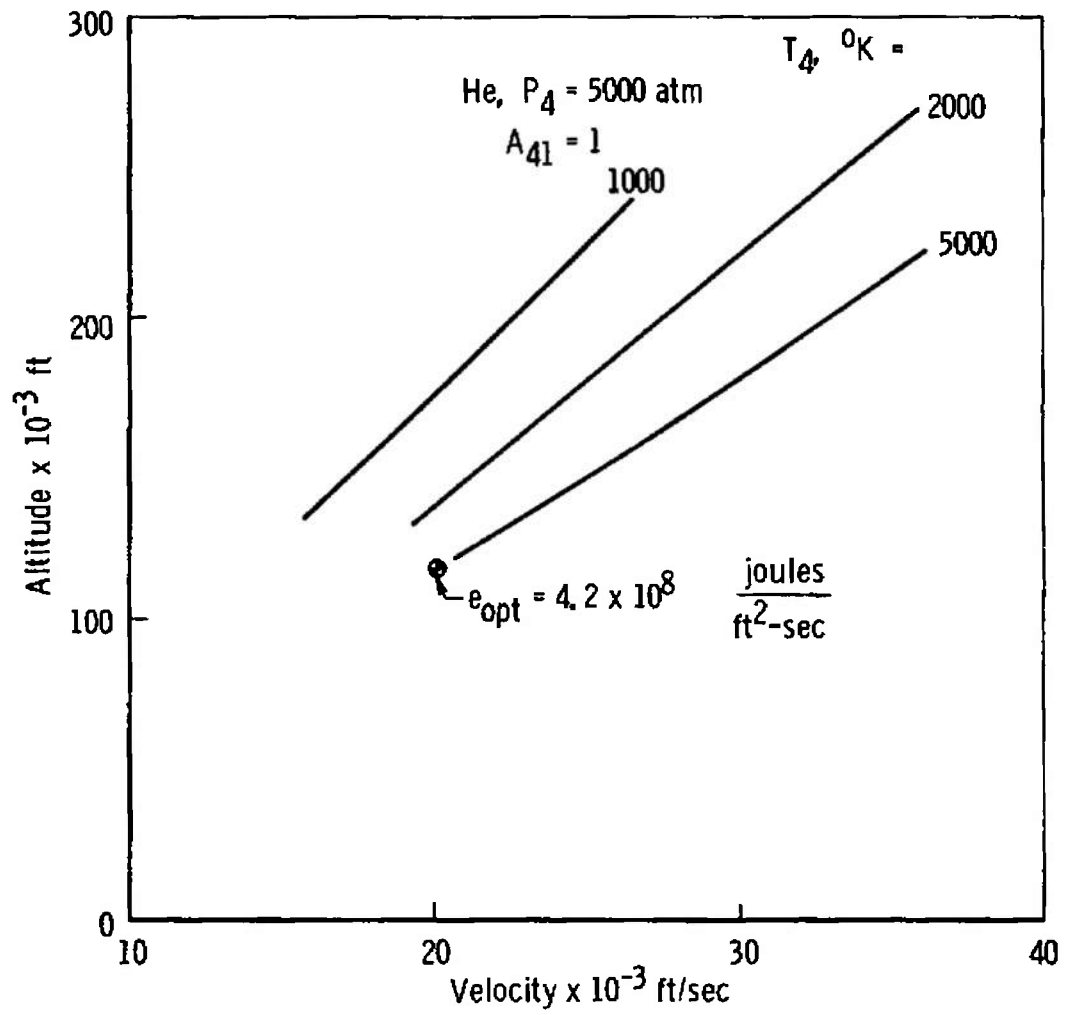
Fig. 6 Driver Requirements



b.  $A_{6A}/A_6 = 10$   
 Fig. 6 Continued



c.  $A_{6A}/A_6 = 100$   
 Fig. 6 Continued



d.  $A_{6A}/A_6 = 1000$

Fig. 6 Concluded

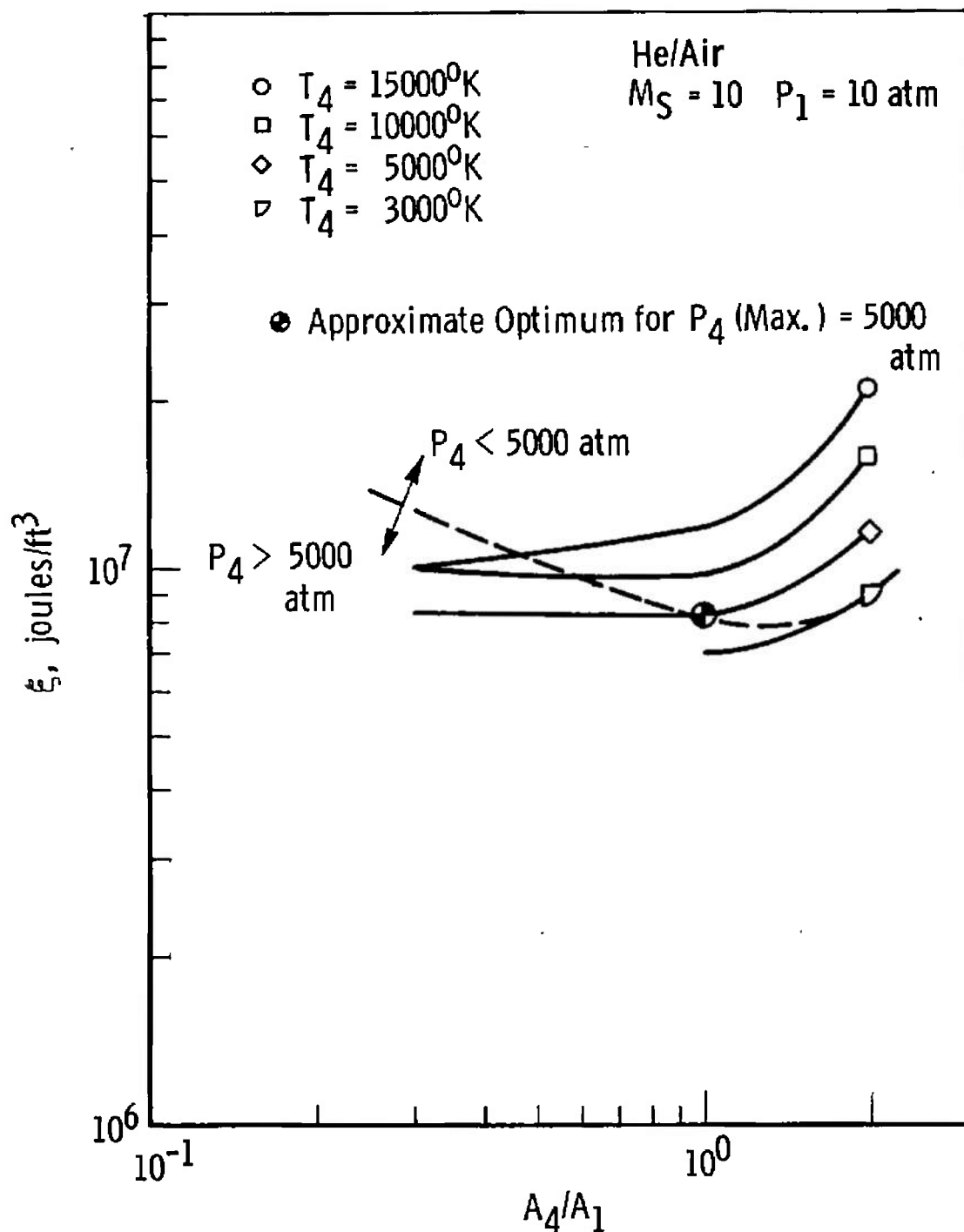


Fig. 7 Optimization of Driver



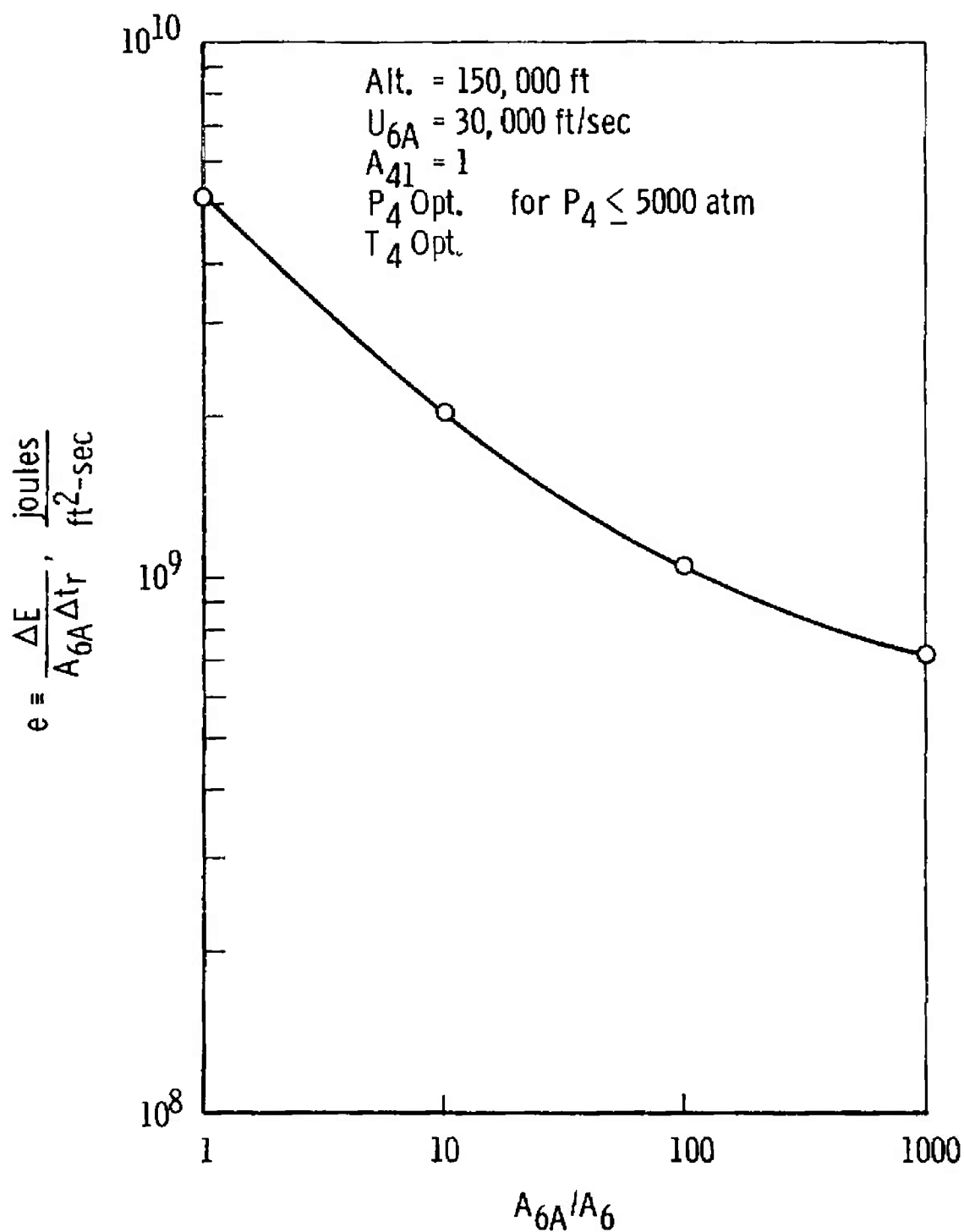


Fig. 8 Comparison of Driver Energy Requirements

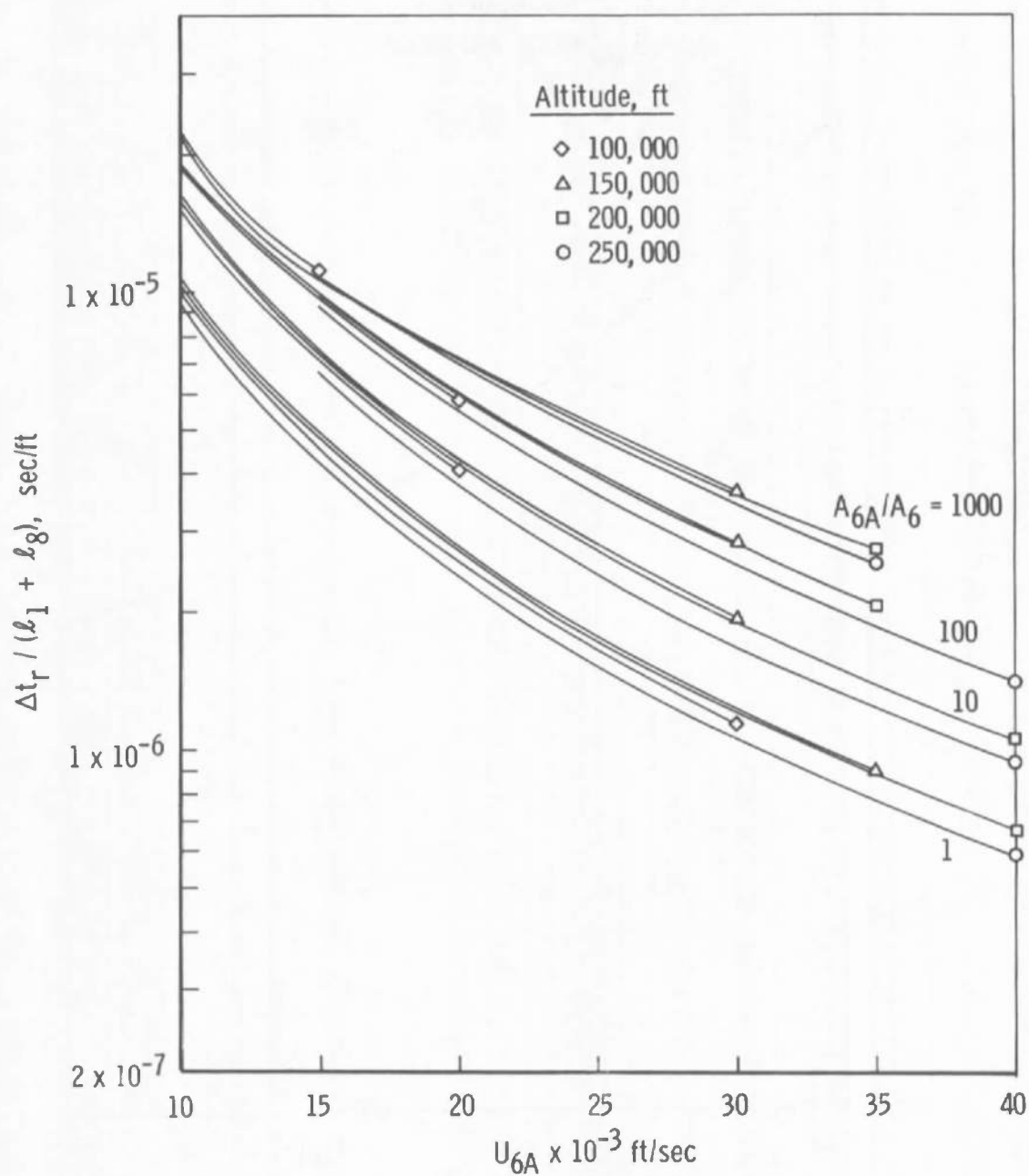


Fig. 9 Ideal Run Time

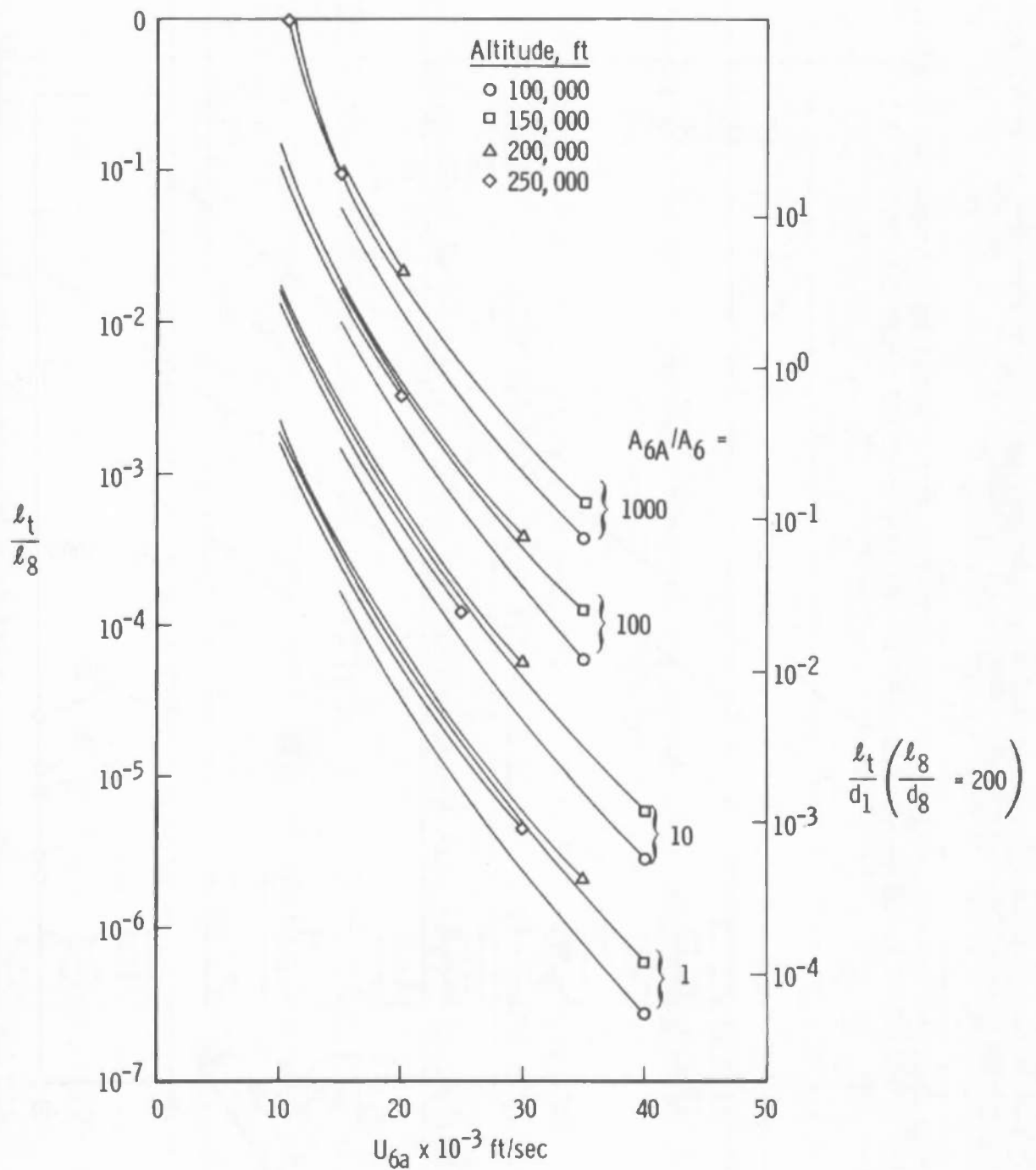


Fig. 10 Proximity of Test Gas to Secondary Diaphragm - Expansion Tunnel

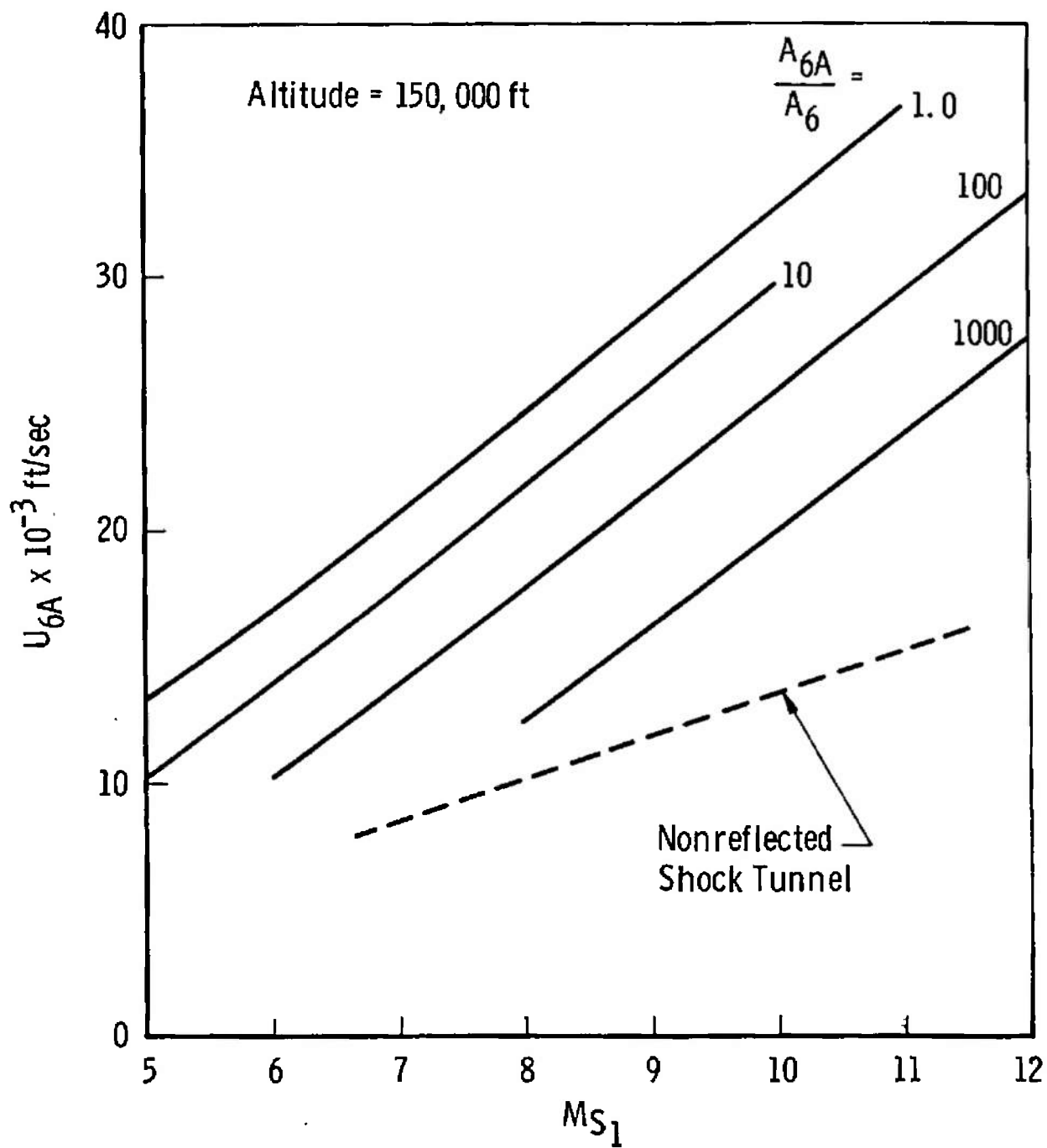


Fig. 11 Loss in Performance Caused by Steady Expansion

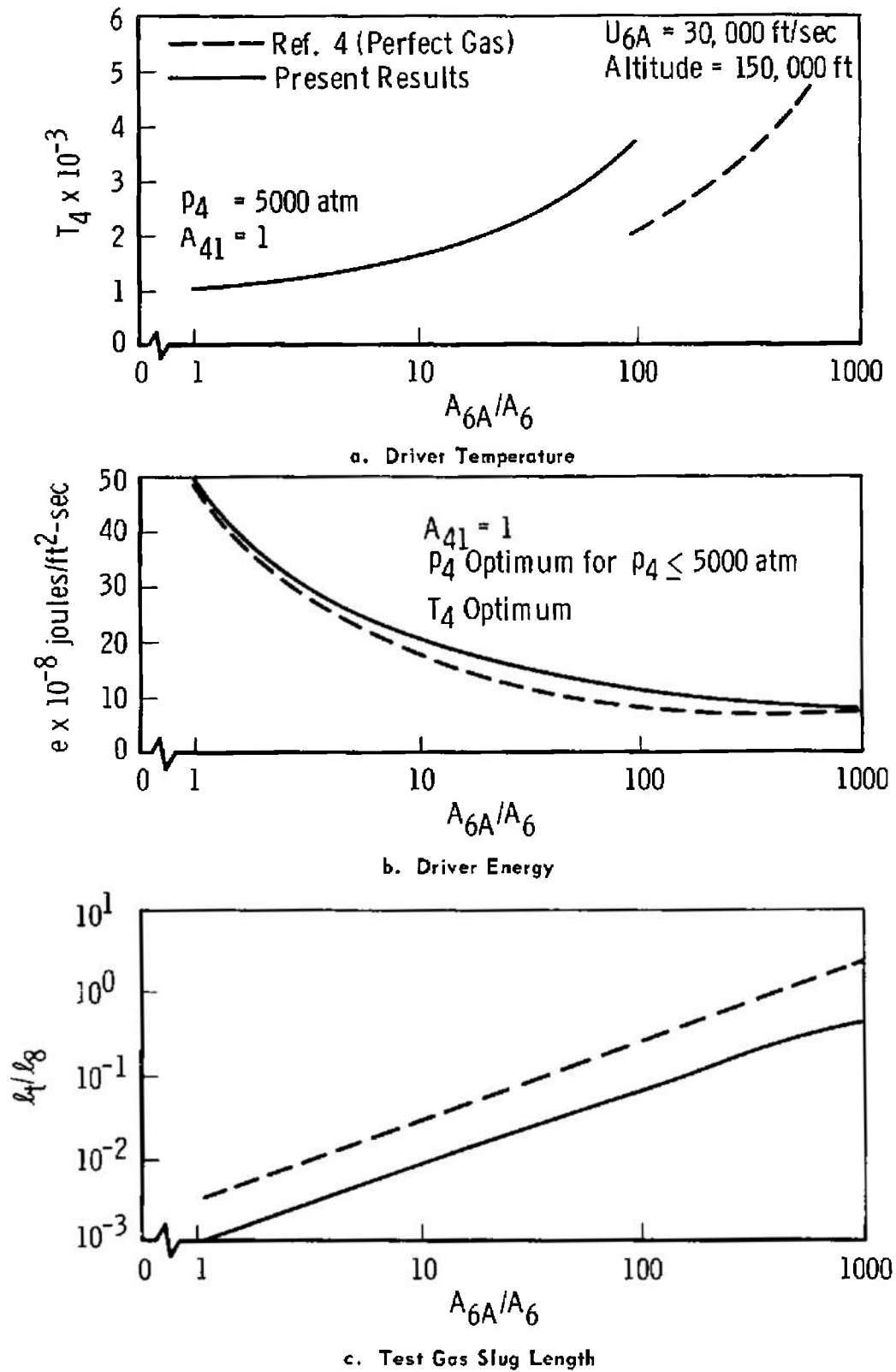


Fig. 12 Variation of Some Critical Parameters with Expansion Area Ratio

**TABLE I**  
**EXPANSION TUNNEL PROGRAM - INPUT AND OUTPUT DATA**

**INPUT DATA**

|          |               |              |       |
|----------|---------------|--------------|-------|
| $U_{6A}$ | Ft (Altitude) | $A_{6A}/A_6$ | $T_1$ |
|----------|---------------|--------------|-------|

**OUTPUT DATA**

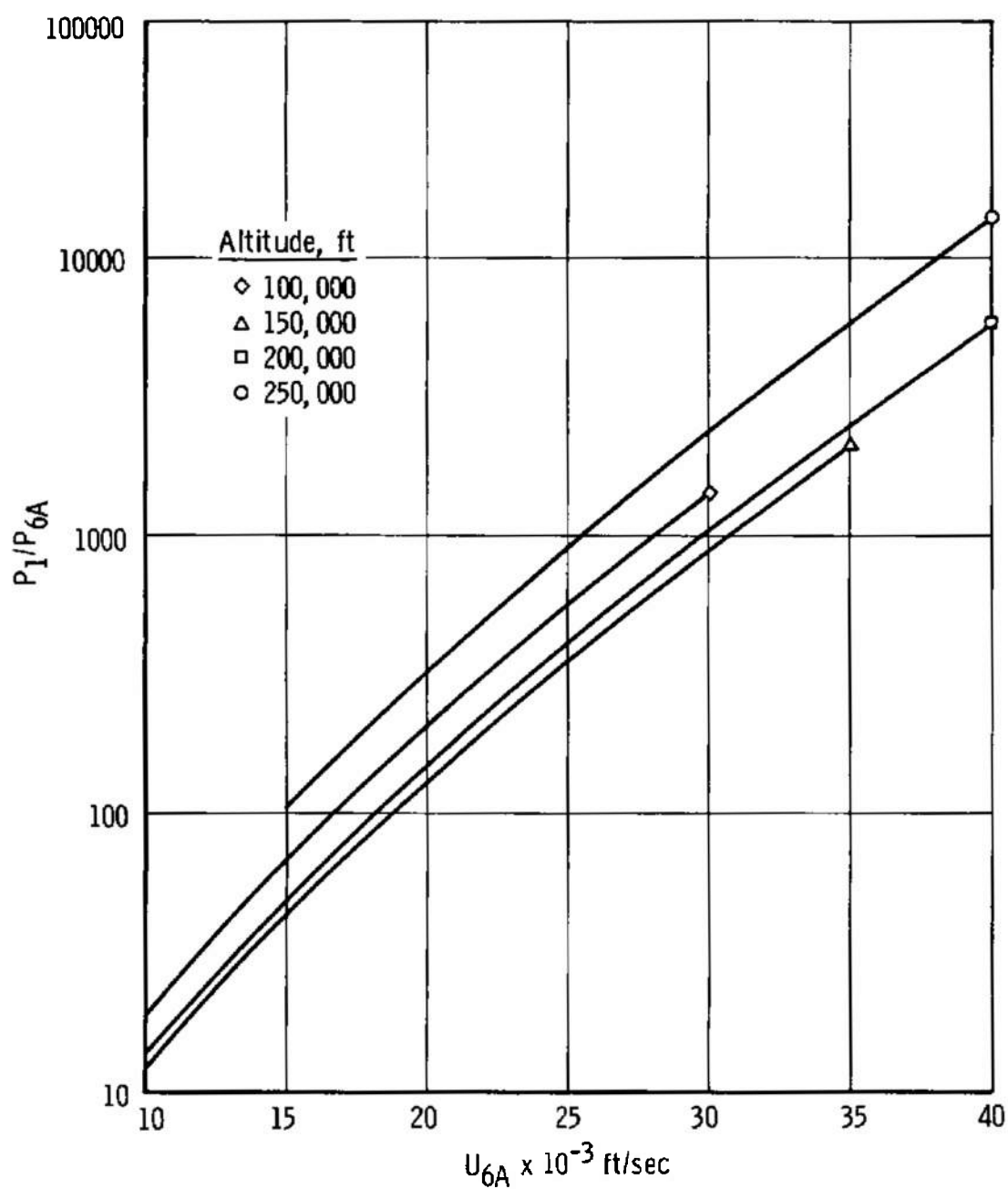
|          |           |           |                 |                     |          |          |
|----------|-----------|-----------|-----------------|---------------------|----------|----------|
| $p_1$    | $U_{S_1}$ | $M_{S_1}$ | $\ell_1/\ell_8$ | $\ell_8/\Delta t_r$ |          |          |
| $p_2$    | $U_2$     | $T_2$     | $\rho_2$        | $h_2$               | $a_2$    | $z_2$    |
| $p_6$    | $U_6$     | $T_6$     | $\rho_6$        | $h_6$               | $a_6$    | $z_6$    |
| $p_{6A}$ | $U_{6A}$  | $T_{6A}$  | $\rho_{6A}$     | $h_{6A}$            | $a_{6A}$ | $z_{6A}$ |

## APPENDIX I WORKING GRAPHS

As an aid in expansion tunnel design and operation, some of the more meaningful parameters are presented in the form of working graphs. Charge pressures  $p_1$  and  $p_8$  are presented in Figs. I-1 and I-2, respectively. Pressure in the shock heated region,  $p_2$ , is presented in Fig. I-3. The nondimensional form,  $p/p_{6A}$ , reduces the variation with altitude to that caused by real-gas effects and acoustic velocity variation.

Shock strengths as a function of altitude and test gas velocity are presented in Figs. I-4 and I-5. Here again, the variation with altitude is caused by variation of acoustic velocity,  $a_{6A}$ , and real-gas effects.

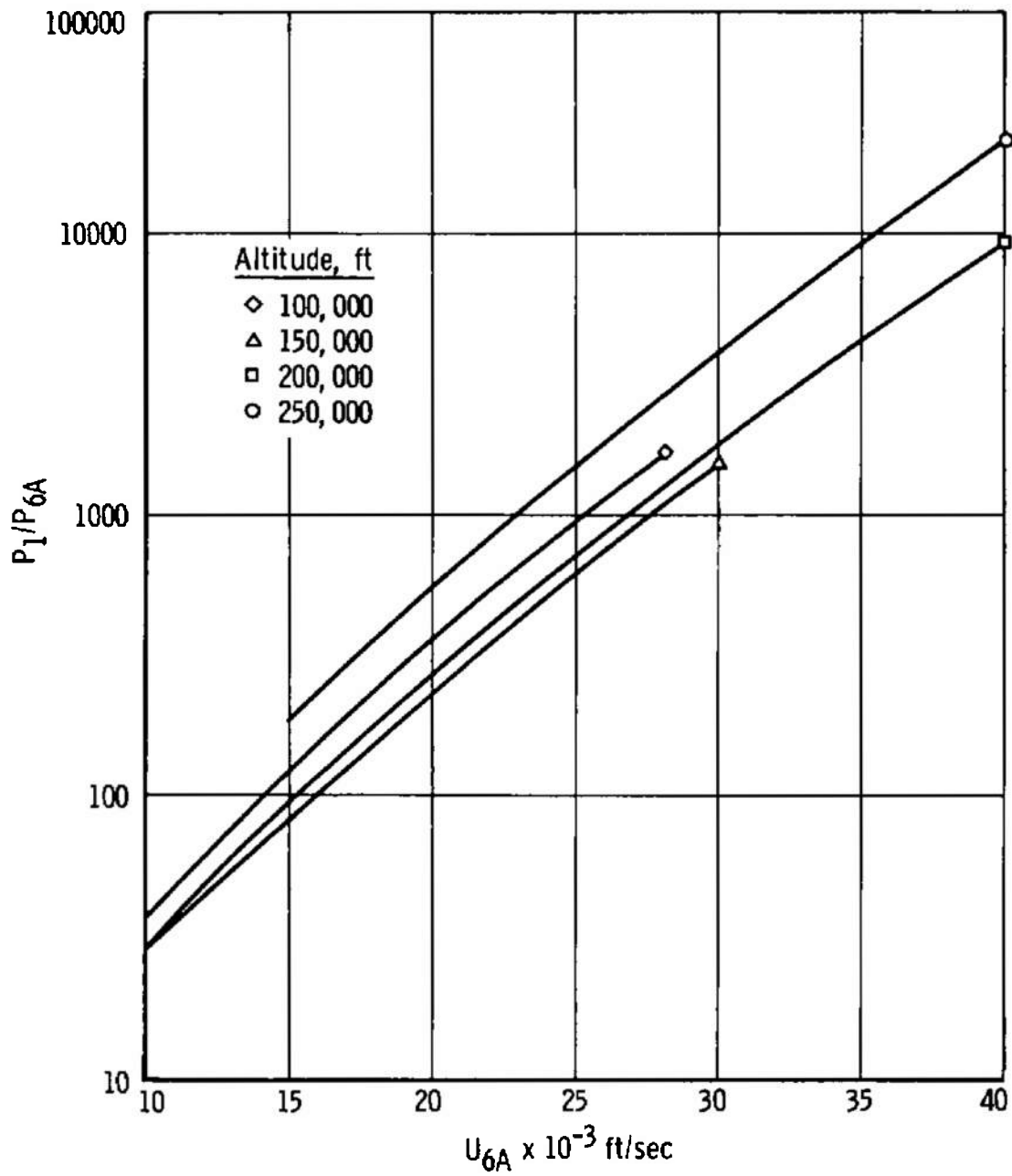
The optimum driven tube length is given as a function of test gas velocity and altitude in Fig. I-6. As noted previously, viscous effects in the driven tube may increase the optimum length considerably. Accelerating tube length per unit run time is shown in Fig. I-7. The effects of viscosity on run time have not been assessed, even qualitatively, to date.



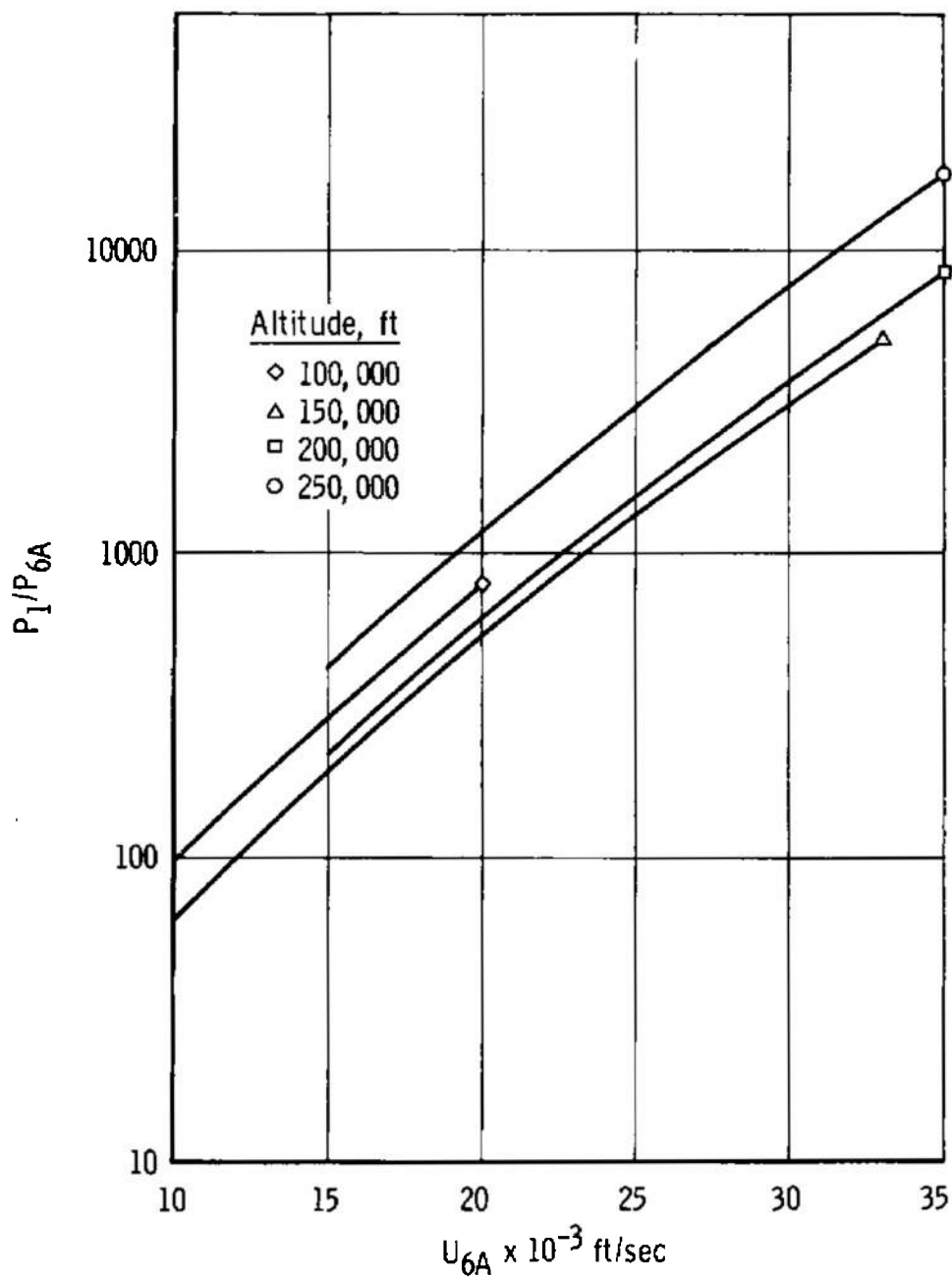
a.  $A_{6A}/A_6 \approx 1.0$

Fig. 1-1 Charge Pressure -  $P_1$

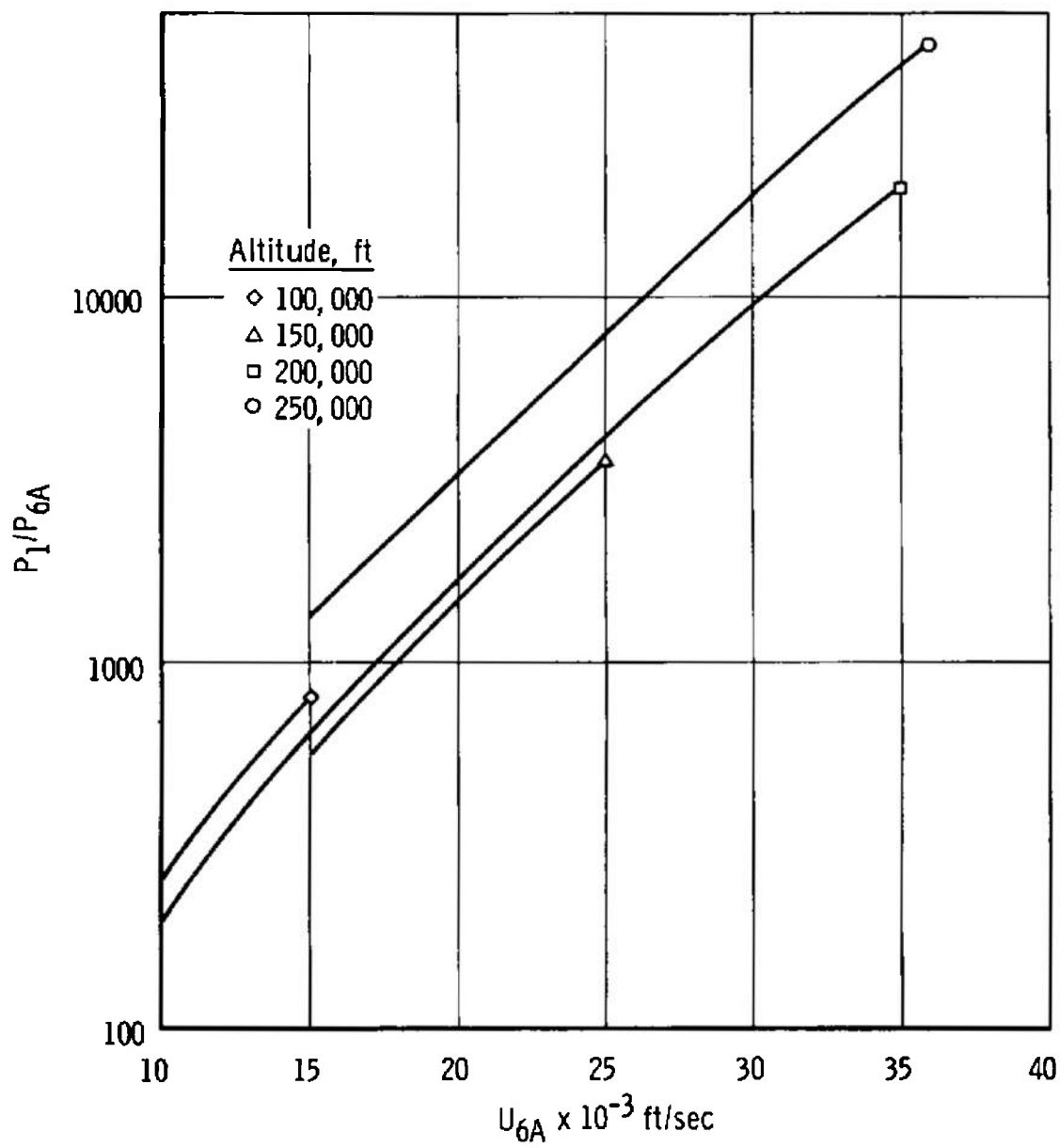




b.  $A_{6A}/A_6 = 10$   
 Fig. 1-1 Continued

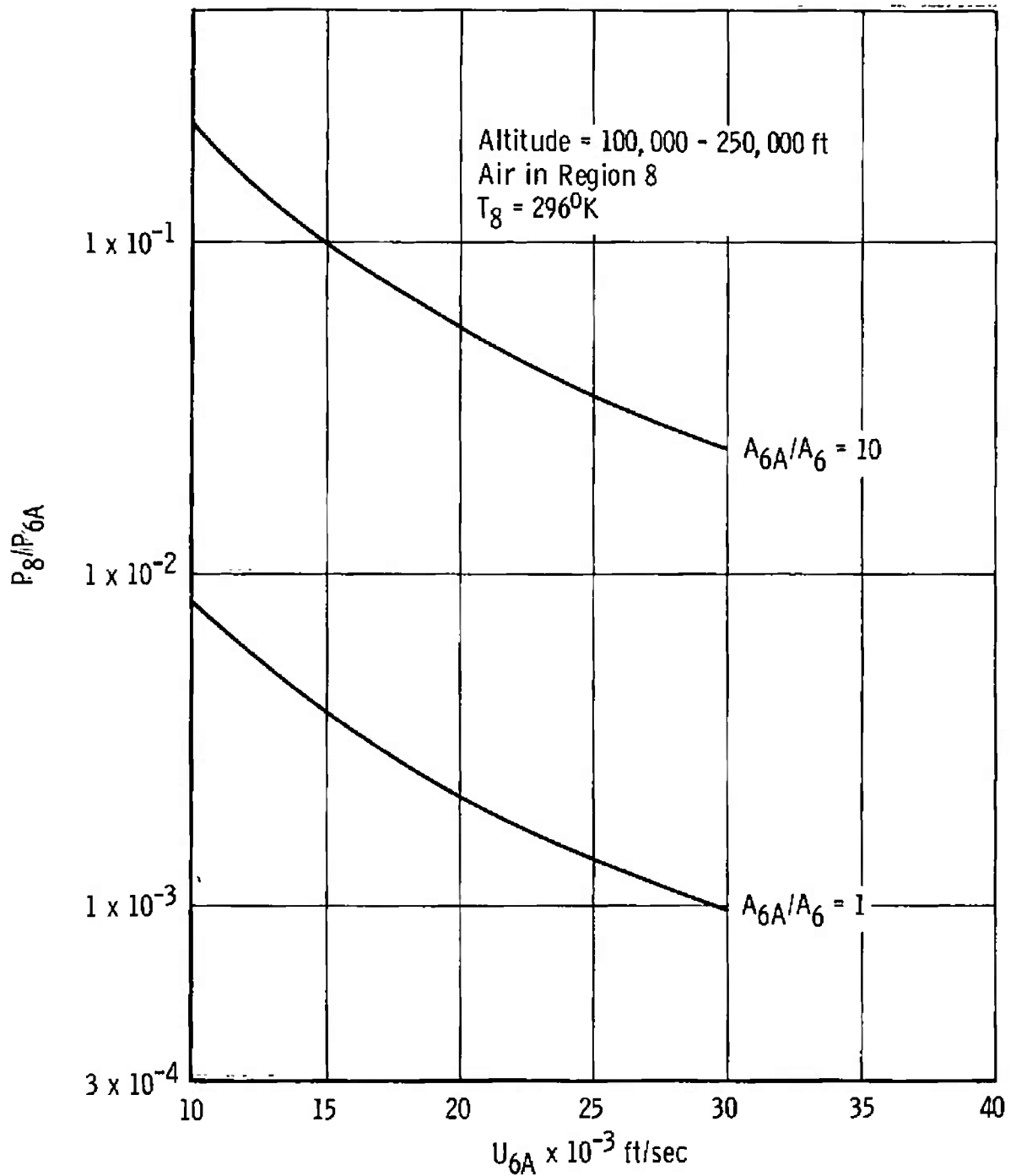


c.  $A_{6A}/A_6 = 100$   
 Fig. 1-1 Continued

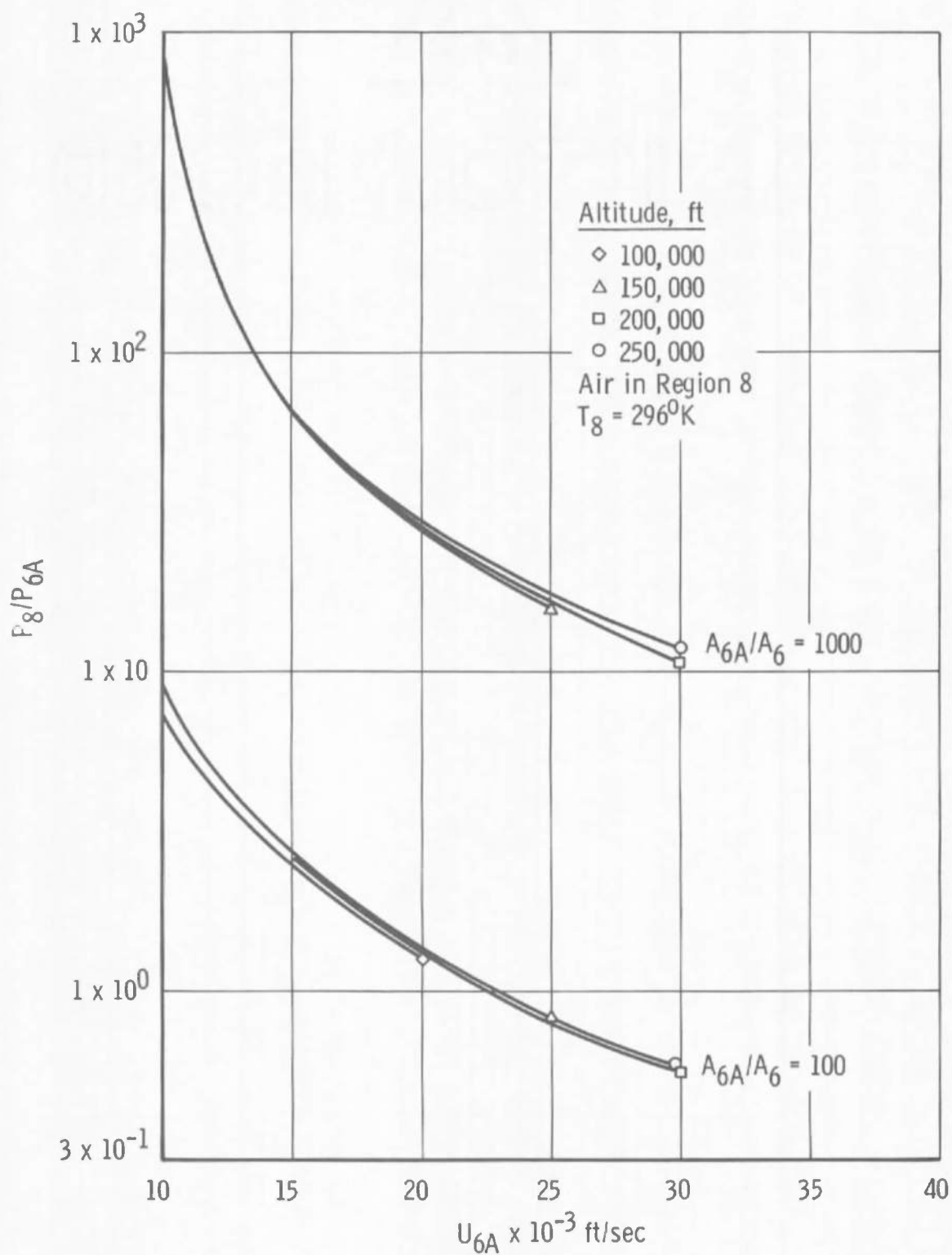


d.  $A_{6A}/A_6 = 1000$

Fig. 1-1 Concluded

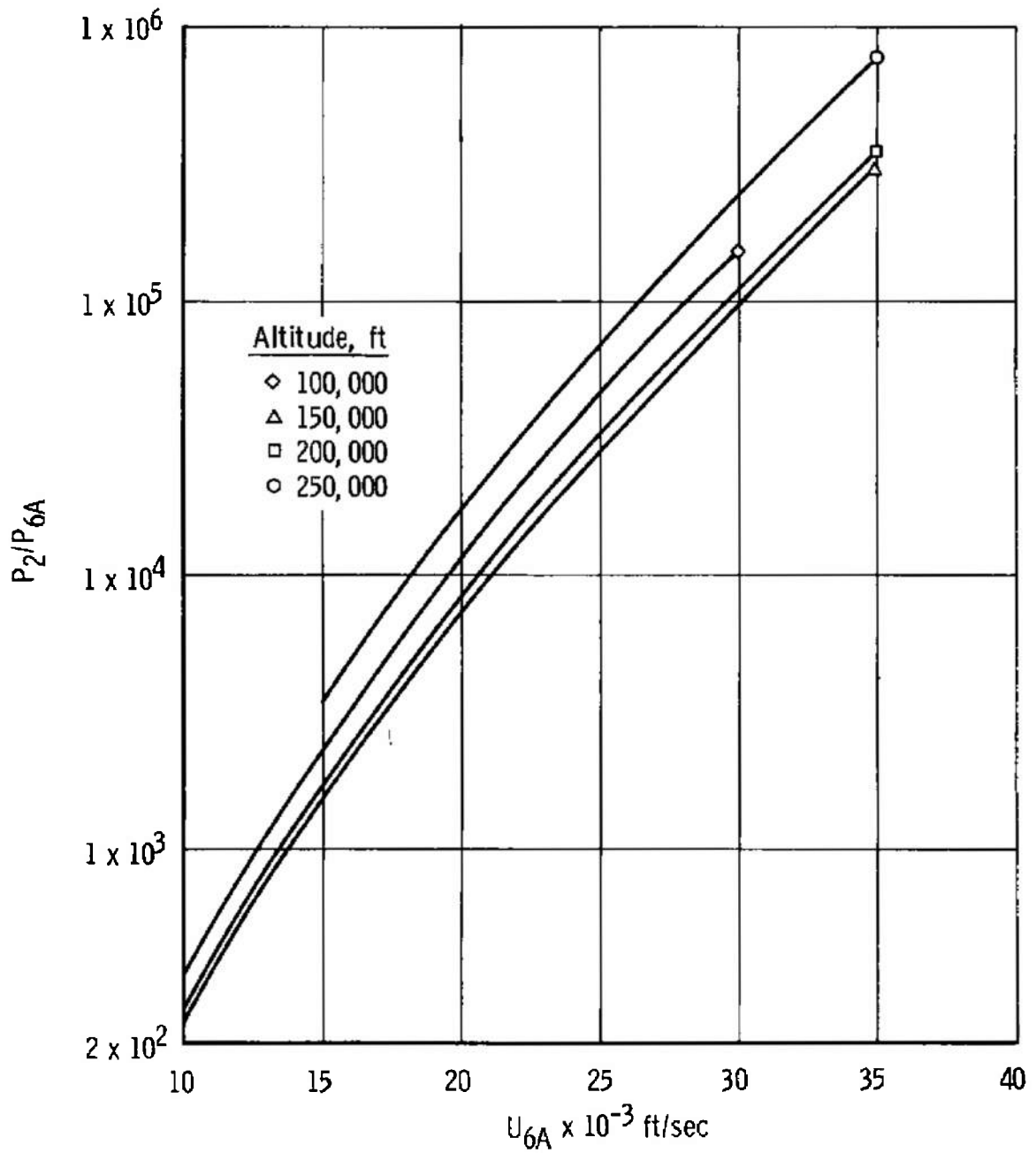


a.  $A_{6A}/A_6 = 10$  and  $A_{6A}/A_6 = 1$   
Fig. 1-2 Charge Pressure -  $P_8$



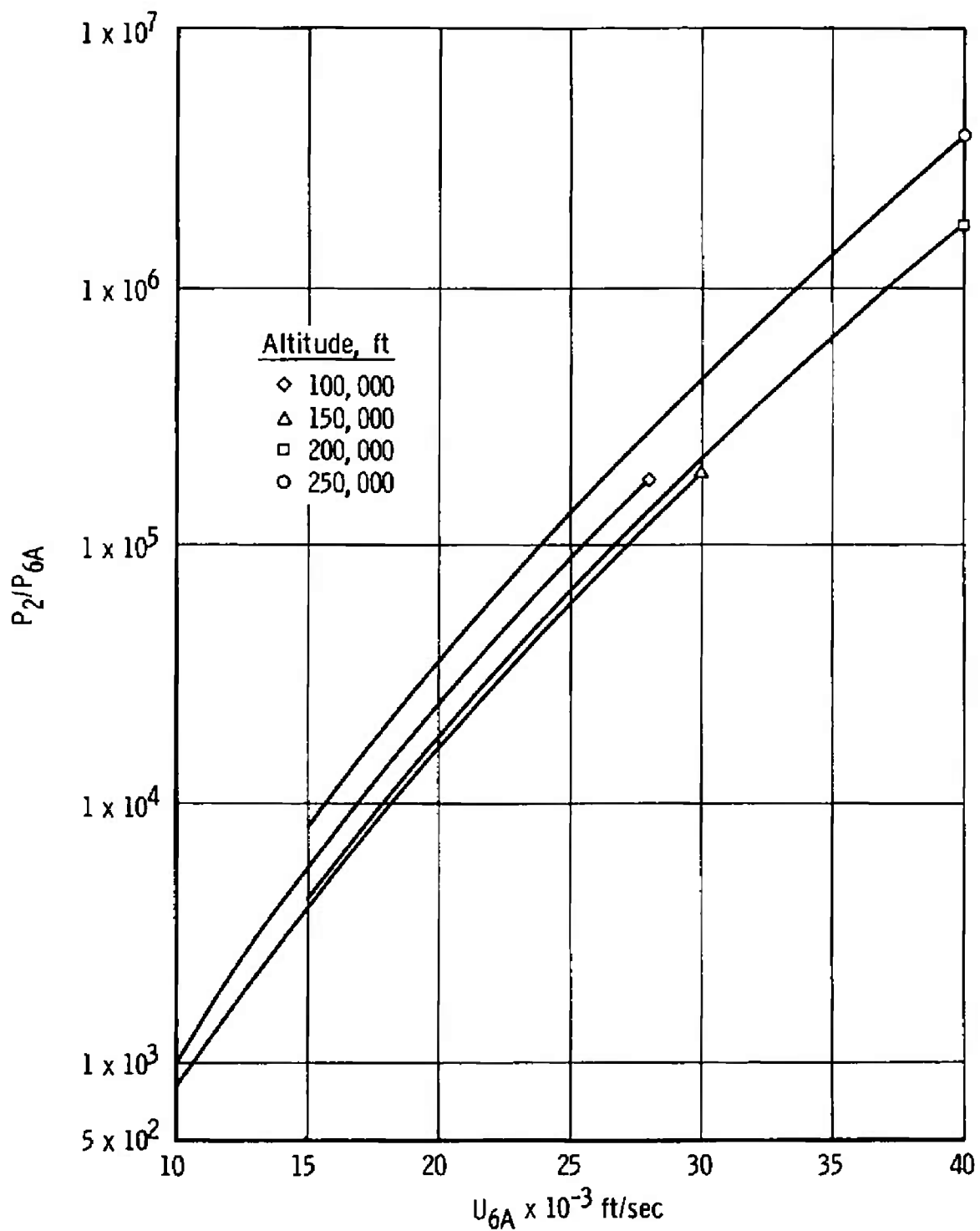
b.  $A_{6A}/A_6 = 1000$  and  $A_{6A}/A_6 = 100$

Fig. 1-2 Concluded



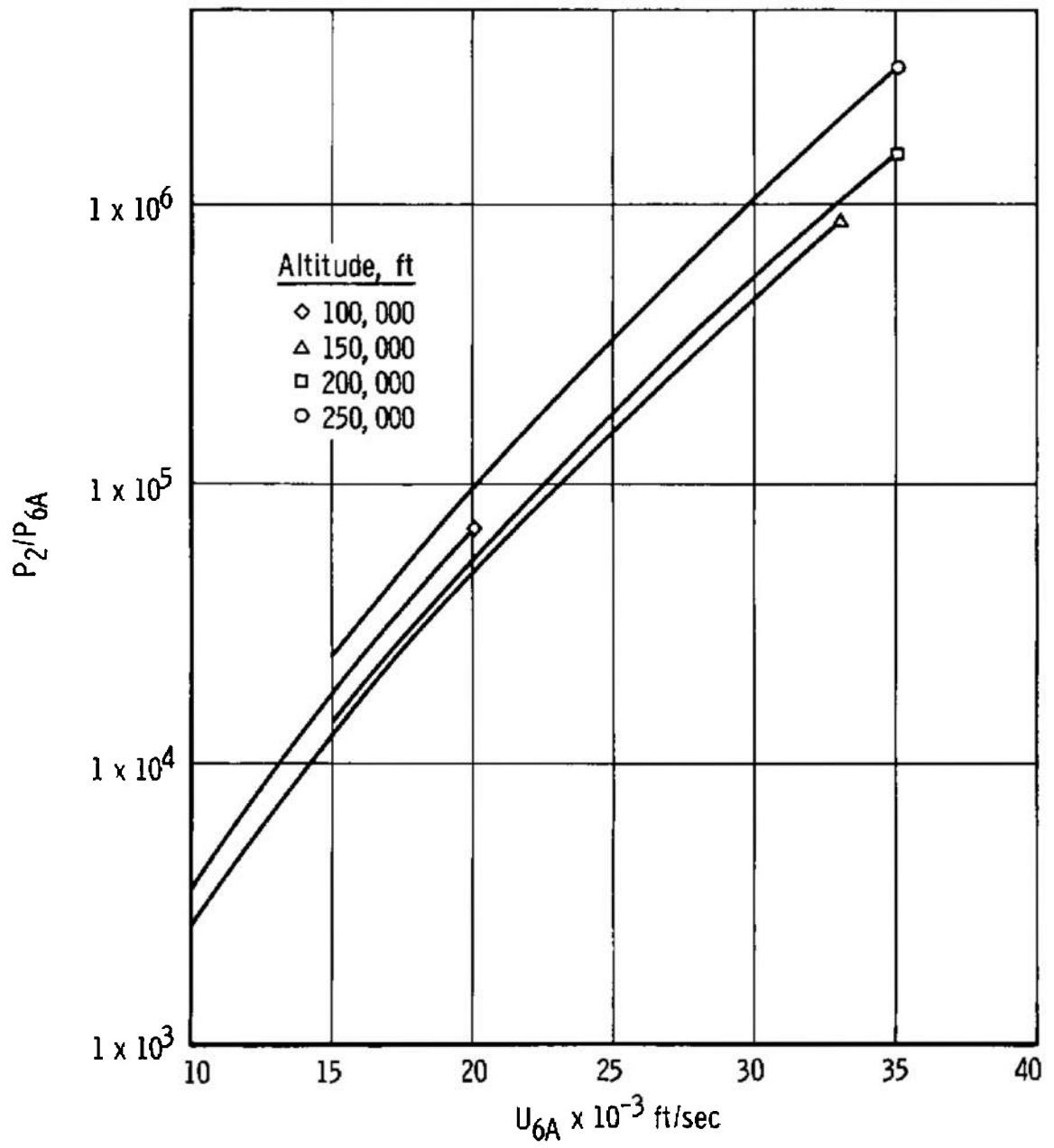
a.  $A_{6A}/A_6 = 1.0$

Fig. I-3 Pressure behind Primary Shock in Driven Tube -  $P_2$



b.  $A_{6A}/A_6 = 10$

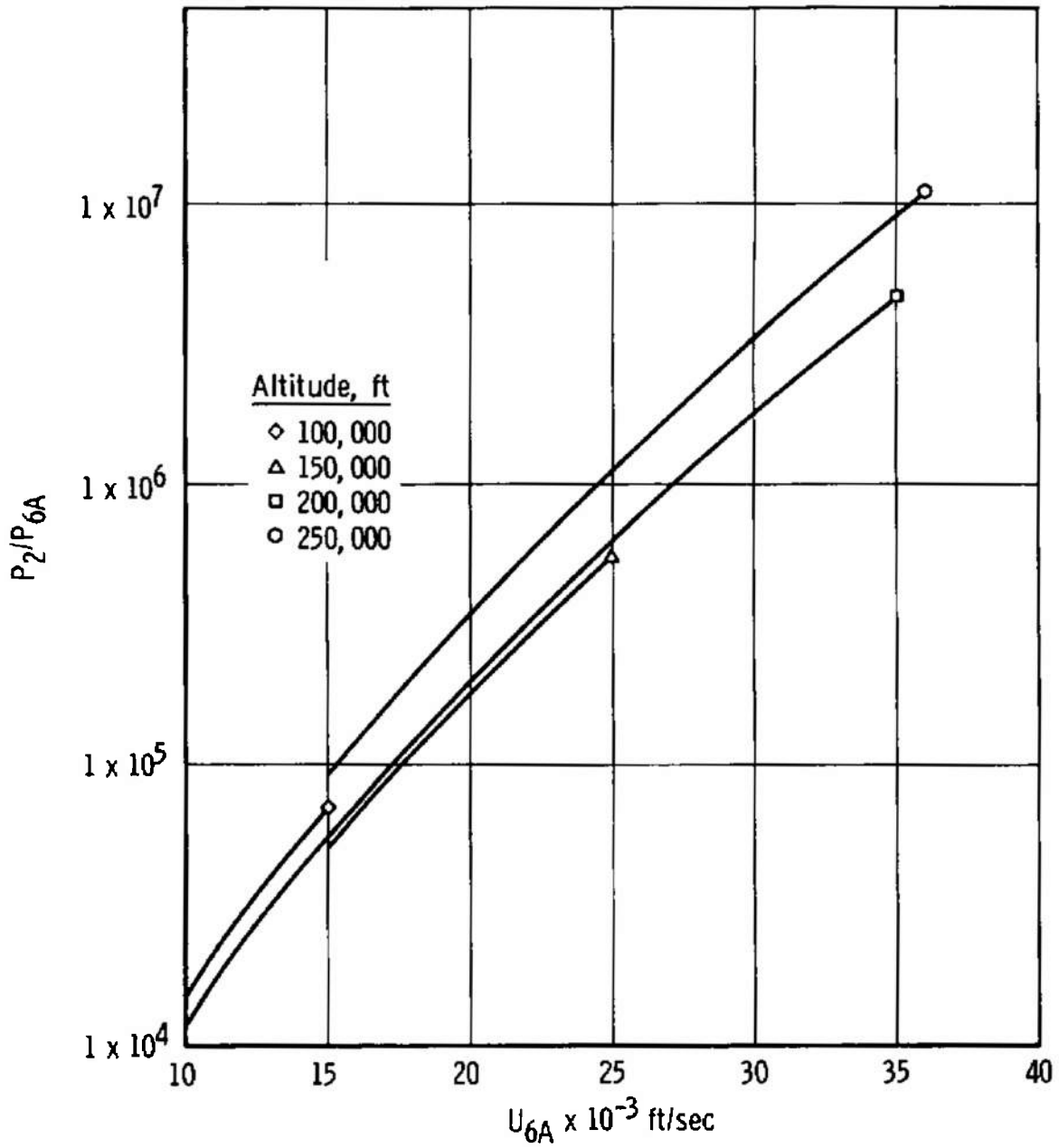
Fig. I-3 Continued



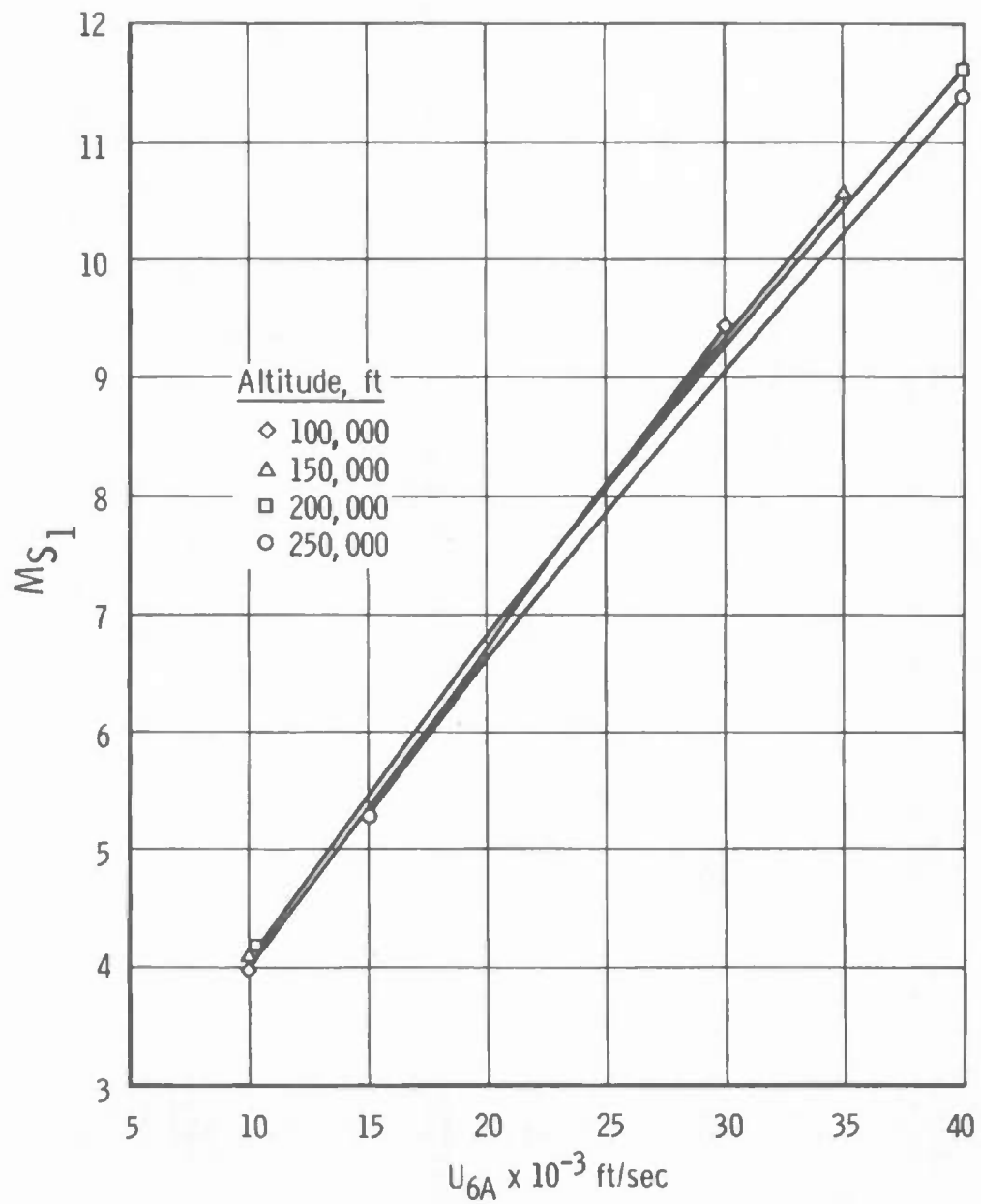
c.  $A_{6A}/A_5 = 100$

Fig. I-3 Continued



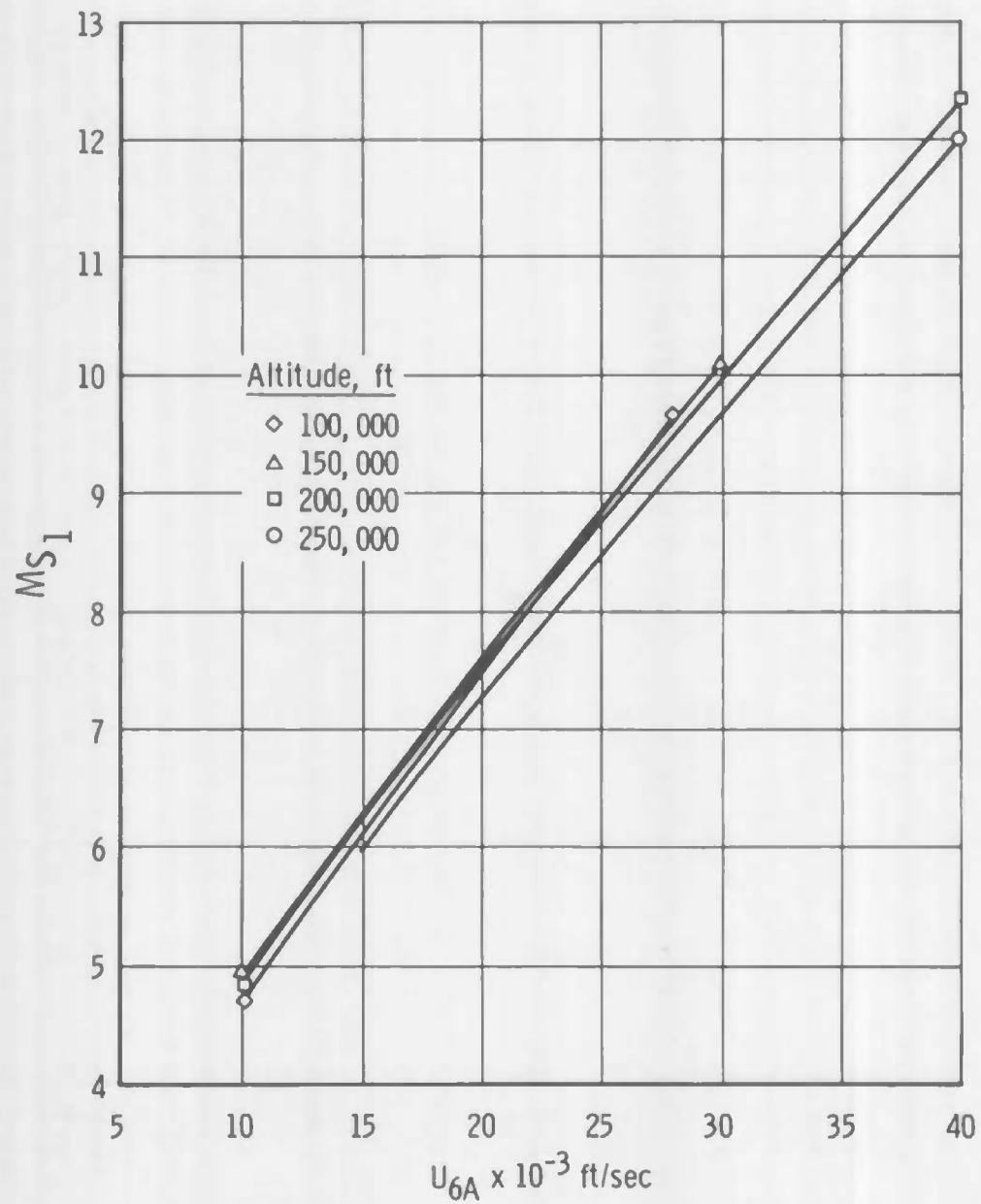


d.  $A_{6A}/A_6 = 1000$   
Fig. 1-3 Concluded

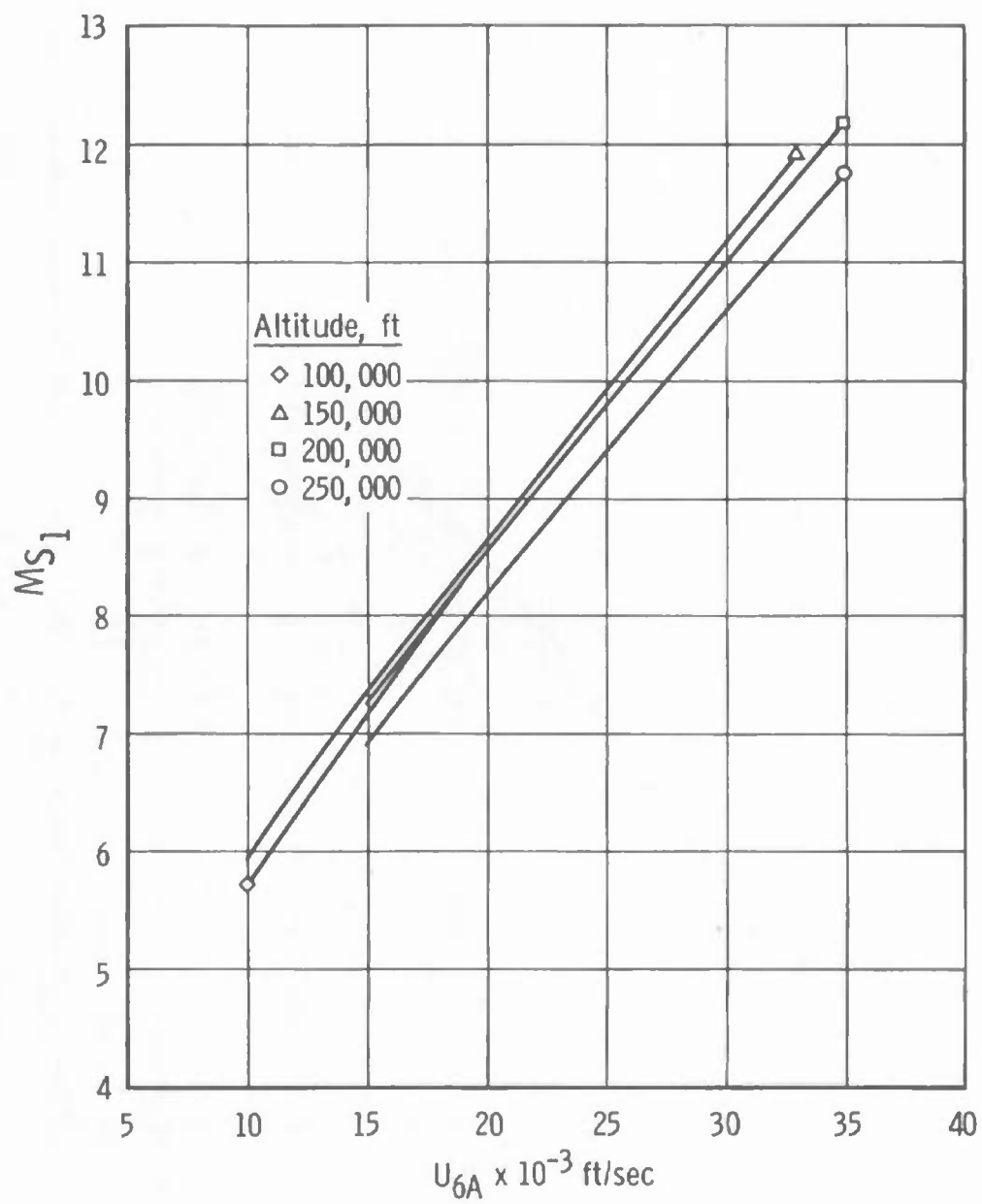


a.  $A_{6A}/A_6 = 1.0$

Fig. I-4 Primary Shock Strength Driven Tube -  $MS_1$

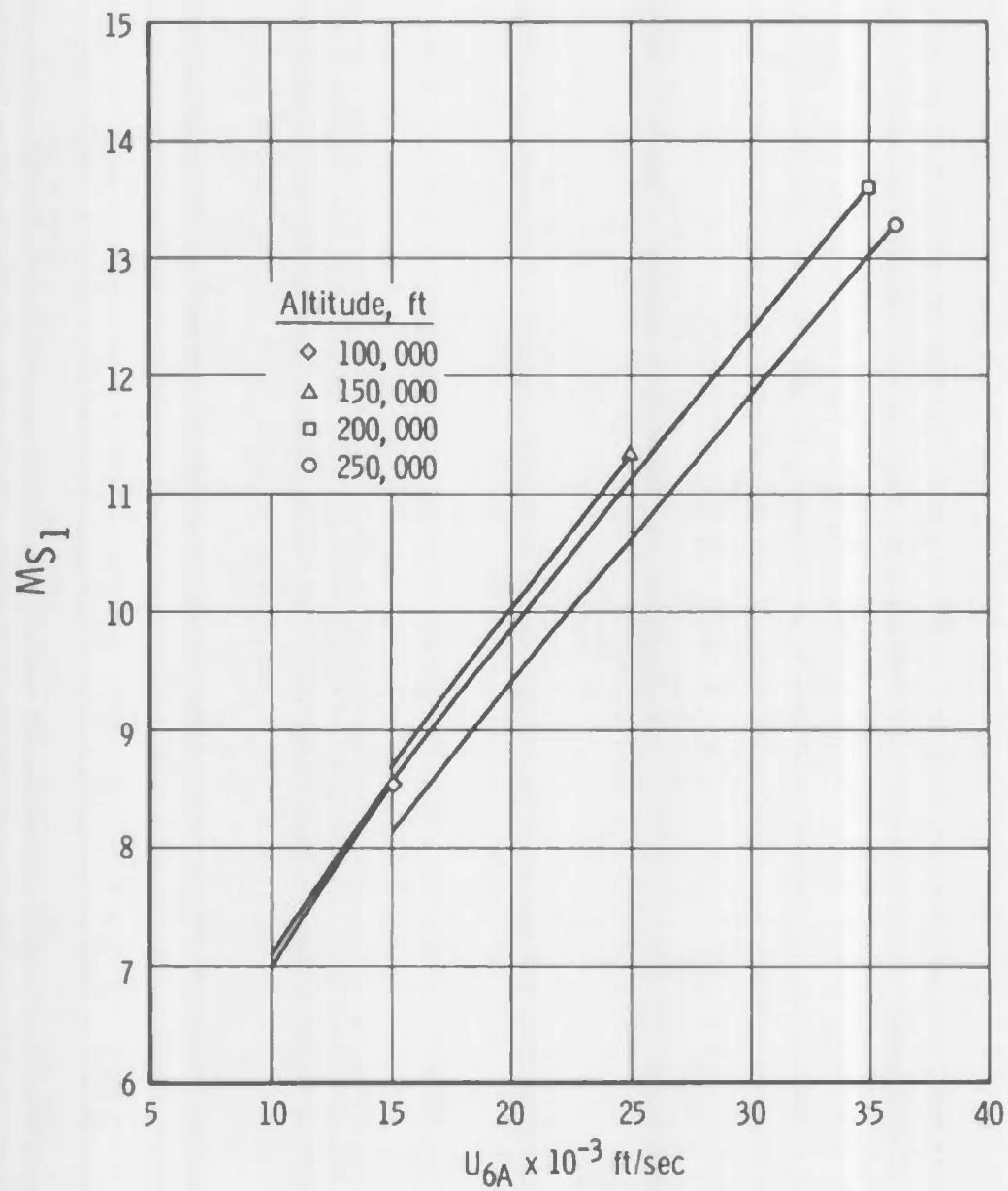


b.  $A_{6A}/A_6 = 10$   
Fig. I-4 Continued



c.  $A_{6A}/A_6 = 100$

Fig. I-4 Continued



d.  $A_{6A}/A_6 = 1000$   
Fig. I-4 Concluded

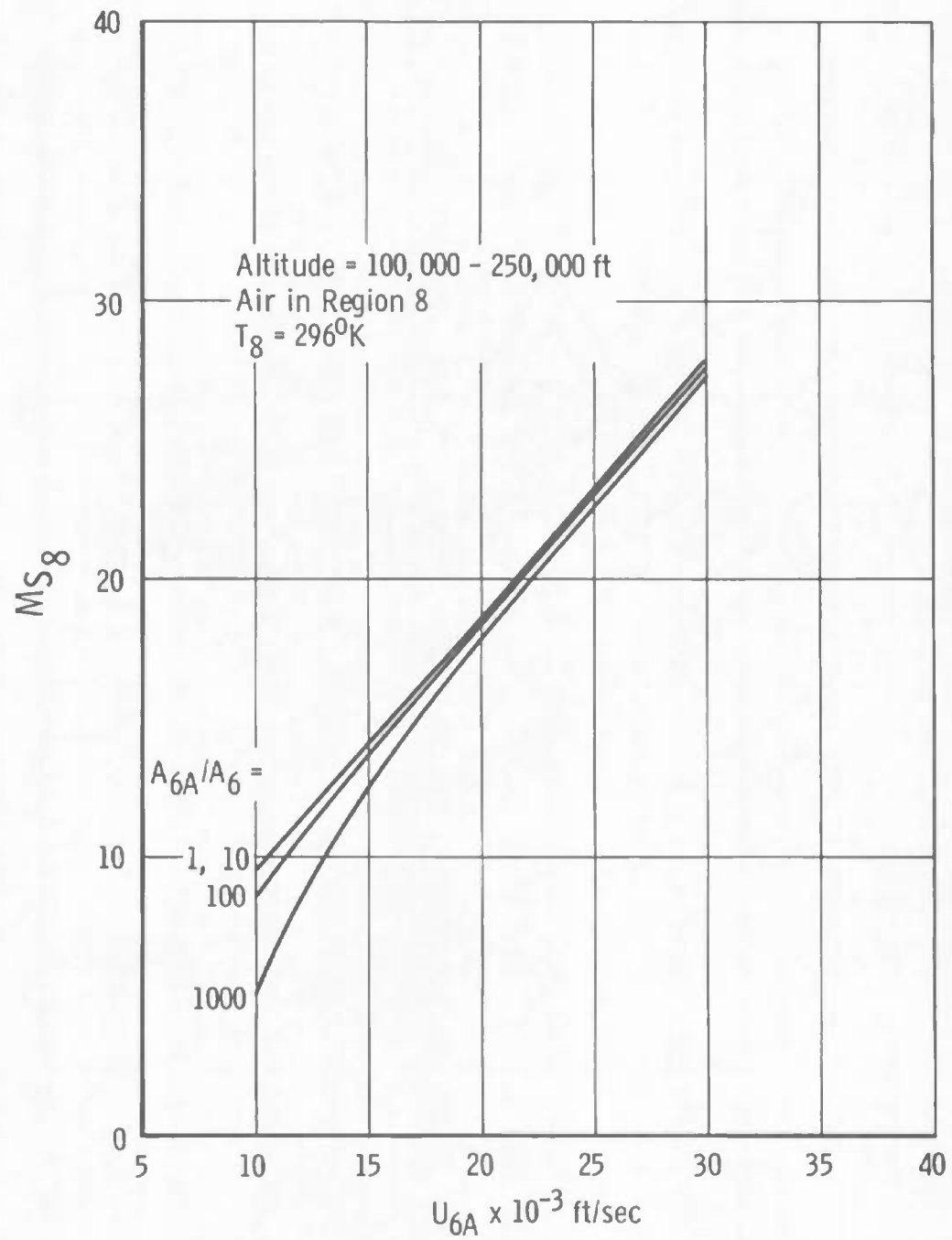


Fig. 1-5 Primary Shock Strength – Accelerating Tube,  $MS_8$

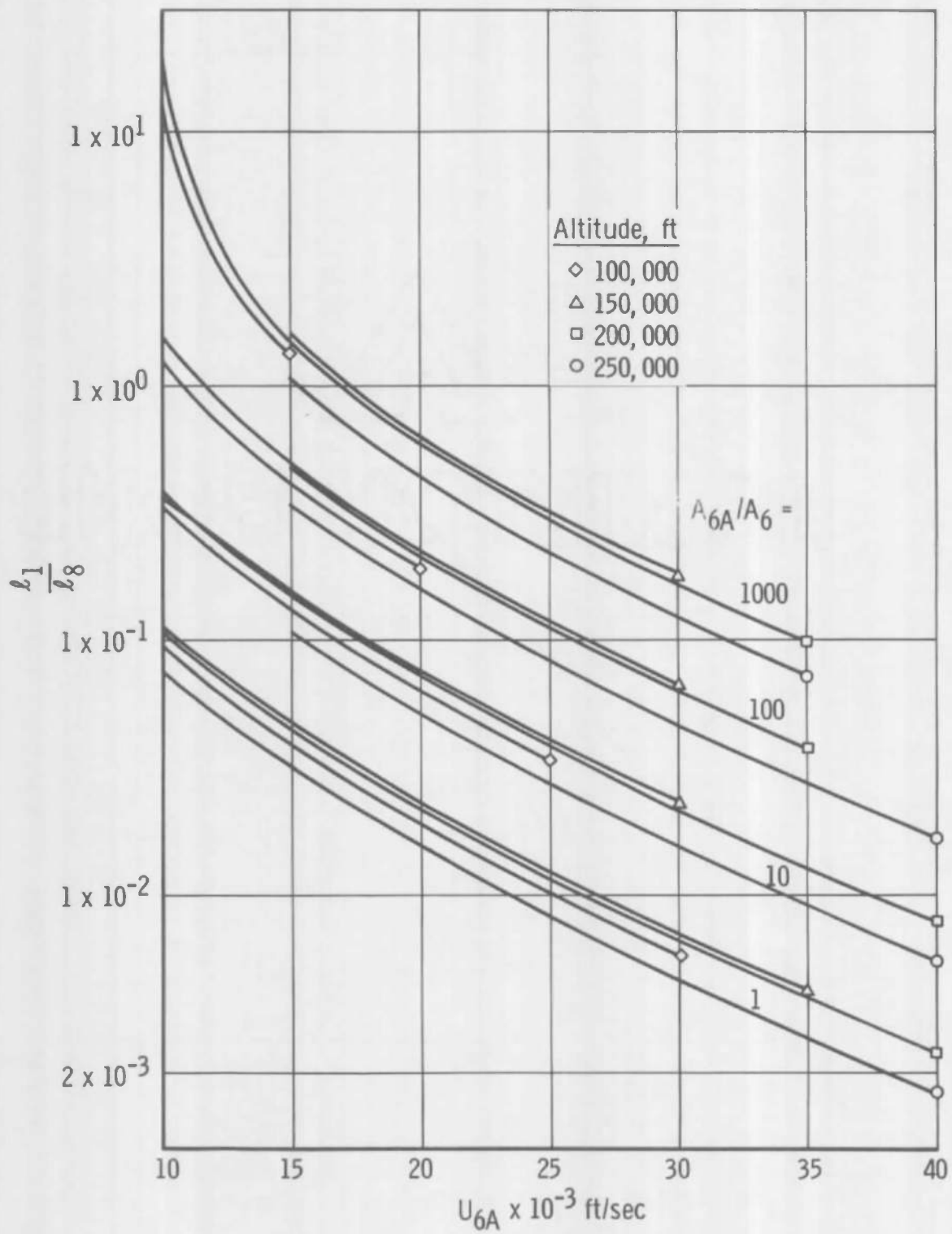


Fig. I-6 Driven Tube/Accelerating Tube Length Ratio

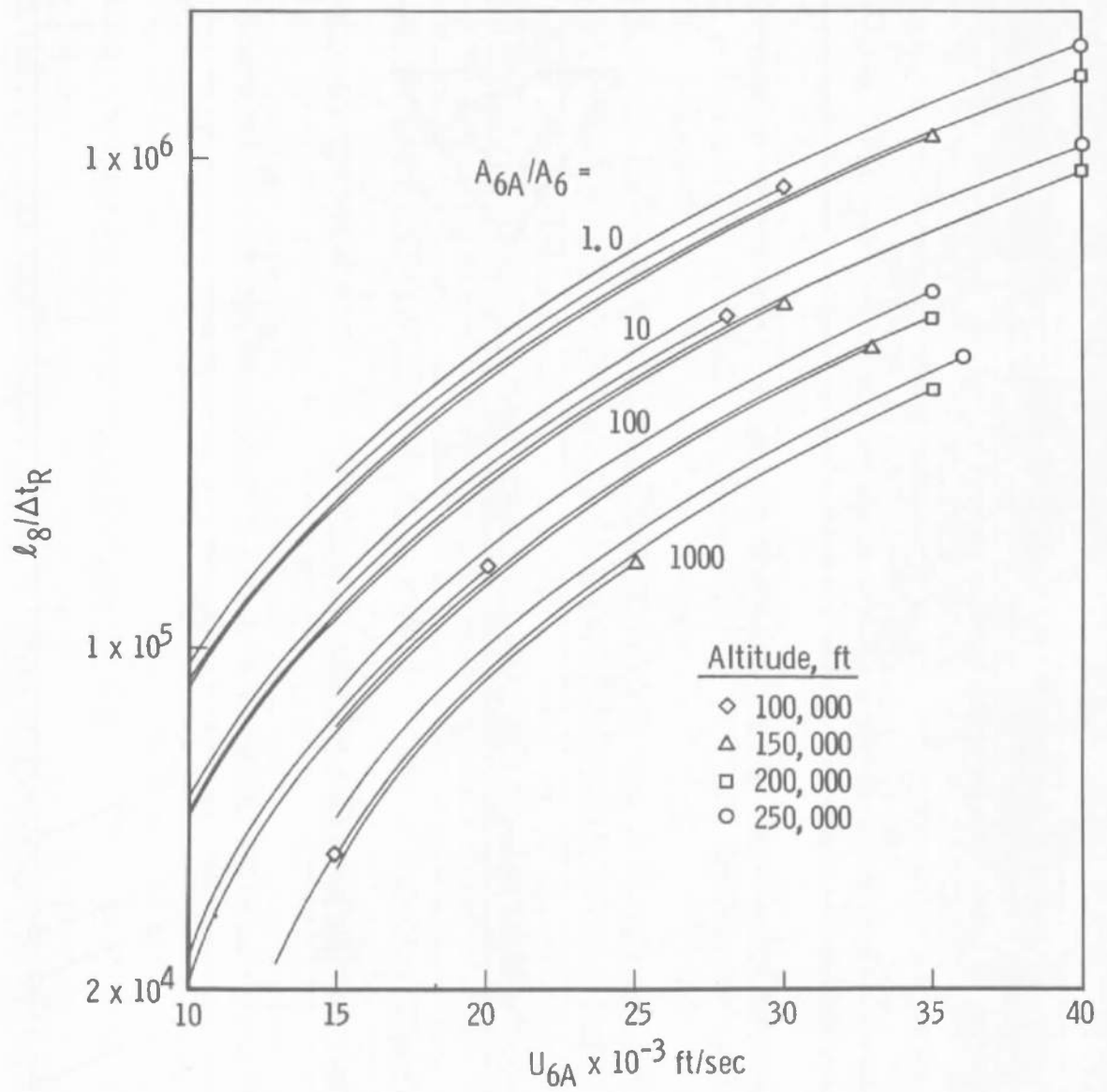


Fig. 1-7 Accelerating Tube Length Per Unit Run Time



## APPENDIX II

### DERIVATION OF EQUATIONS USED IN THE COMPUTER PROGRAM

#### SHOCK CROSSING EQUATIONS

For a plane shock wave moving into a quiescent gas, the equations for the conservation of mass, momentum, and energy, and the equation of state are, respectively,

$$\rho_2 (U_{S1} - U_2) = \rho_1 U_{S1} \quad (\text{II-1})$$

$$p_2 + \rho_2 (U_{S1} - U_2)^2 = p_1 + \rho_1 U_{S1}^2 \quad (\text{II-2})$$

$$h_2 + 1/2 (U_{S1} - U_2)^2 = h_1 + 1/2 U_{S1}^2 \quad (\text{II-3})$$

$$p_1 = \rho_1 R T_1 \quad (\text{II-4})$$

These four equations will be used to determine  $\rho_1$ ,  $U_{S1}$ ,  $p_1$ , and  $U_2$ .

Eliminating  $U_2$  and  $U_{S1}^2$  from Eqs. (II-1), (II-2), and (II-3) gives<sup>9</sup>

$$(p_2/p_1 - 1) - \frac{p_1 R T_a}{\rho_1} = \frac{(h_2/h_1 - 1) 2h_1}{1 + \rho_1/\rho_2} \quad (\text{II-5})$$

Now, eliminating  $p_1$  between Eqs. (II-4) and (II-5), one gets, after some manipulation,

$$\rho_1^2 + \rho_1 \left[ \frac{2\rho_2 (h_2 - h_1)}{R T_1} + \rho_2 - \frac{p_2 T_a}{T_1} \right] - \frac{p_2 \rho_2 T_a}{T_1} = 0 \quad (\text{II-6})$$

The positive sign in the quadratic formula corresponding to the above equation yields the desired value of  $\rho_1$ . Equations (II-3), (II-4), and (II-1) can be written

$$U_{S1} = \left\{ 2(h_2 - h_1) / \left[ 1 - (\rho_1/\rho_2)^2 \right] \right\}^{1/2} \quad (\text{II-7})$$

---

<sup>9</sup>Introducing the nondimensional quantities of  $p$  in atm and  $\rho$  in amagats.

$$p_1 = \rho_1 T_1 / T_a \quad (\text{II-8})$$

$$U_2 = U_{S_1} (1 - \rho_1 / \rho_2) \quad (\text{II-9})$$

Use is made of Eqs. (II-6), (II-7), (II-8), and (II-9) in Appendix III.

### LENGTH EQUATIONS

Equation (II-3), also appearing in Appendix III, will now be derived. Reference to Fig. II-1 will aid in the understanding of the following development. Hence, from Fig. II-1 it follows that

$$\ell_1 / \Delta t_y = a_2 U_{S_1} / (U_{S_1} - U_2) \quad (\text{II-10})$$

$$\ell_8 / \Delta t_z = U_6 - a_6 \quad (\text{II-11})$$

For the unsteady expansion between regions 2 and 6, the following equations are valid:

$$\frac{dt}{d\ell} = \frac{1}{U + a} \quad (\text{II-12})$$

$$\frac{t - \Delta t_x}{\ell} = \frac{1}{U - a} \quad (\text{II-13})$$

$$a dU = -dh \quad (\text{II-14})$$

The parameter  $\ell$  can be eliminated between Eqs. (II-12) and (II-13) by differentiating (Eq. II-13) and putting the result into Eq. (II-12).

$$(U - a) dt + (dU - da)(t - \Delta t_x) = (U + a) dt \quad (\text{II-15})$$

Using Eq. (II-14) to remove  $dU$  from Eq. (II-15) and simplifying gives

$$\frac{2dt}{t - \Delta t_x} + \frac{da}{a} = - \frac{dh}{a^2} \quad (\text{II-16})$$

Integrating Eq. (II-16) between regions 2 and 6 gives

$$\log \left| a_2 (t_2 - \Delta t_x)^2 \right| = - \int_{h_6}^{h_2} \frac{dh}{a^2} + \log \left| a_6 (t_6 - \Delta t_x)^2 \right| \quad (\text{II-17})$$

Hence,

$$\log \left| \frac{a_2}{a_6} \left( \frac{\Delta t_y}{\Delta t_z} \right)^2 \right| = - \frac{\text{sum}}{a_n}$$

where

$$\text{sum} = \frac{a_a}{r_a T_a} \int_{h_6/R}^{h_2/R} \frac{d(h/R)}{(a/a_a)^2}$$

so that

$$\frac{\Delta t_y}{\Delta t_z} = \left( \frac{a_6}{a_2} \right)^{1/2} \exp(-0.5 \text{ sum}/a_a) \quad (\text{II-18})$$

Combining Eqs. (II-10), (II-11), and (II-18) gives

$$\frac{\ell_1}{\ell_8} = \frac{1}{(U_6 - a_6) \frac{U_{S1} - U_2}{a_2 U_{S1}} \left( \frac{a_2}{a_6} \right)^{1/2} \exp(0.5 \text{ sum}/a_a)}$$

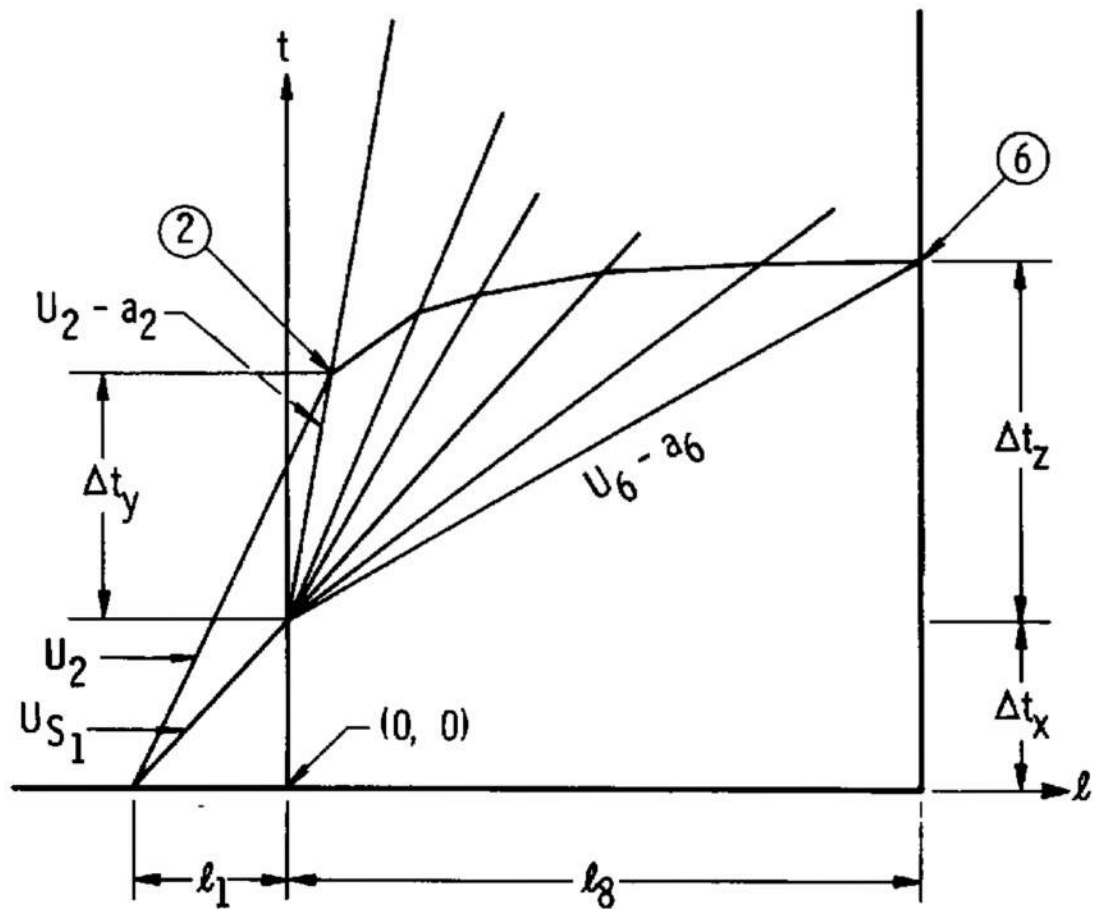
or, in the terminology of Appendix III,

$$\ell_1 = \frac{1}{(U_6 - a_6) y}$$

where

$$y = \frac{U_{S1} - U_2}{a_2 U_{S1}} \left( \frac{a_2}{a_6} \right)^{1/2} \exp(0.5 \text{ sum}/a_a)$$

$$\ell_8 = 1$$



The symbols associated with the above lines represent the reciprocal of the slope of the lines.

Fig. II-1 Wave Diagram Illustrating Nomenclature Used in Calculation of Tube Length

### APPENDIX III COMPUTER PROGRAM

The purpose of this program is to compute certain quantities of interest associated with the unsteady expansion problem among which are the 6 conditions, the 2 conditions, and a characteristic length.

The method by which the above is accomplished basically reduces to solving two separate nonlinear equations by iterative schemes, and evaluating certain integrals by Gaussian mechanical quadrature. The gas properties are retrieved from magnetic tape using a double four-point interpolation.

Program flow is as follows (Figs. III-1 and 2, and Tables III-1, -2, -3, and -4):

- A. SR, if not read in, is computed from T6A and P6A, T6A and P6A being computed as functions of altitude in subroutine ATP.
- B. The remaining 6A conditions are computed from SR and P6A.
- C. An iteration is performed to find  $\rho_6$ . The initial guess is

$$\rho_6 = \rho_{6A} \frac{A_{6A}}{A_6}$$

Given a value of  $\rho_6$ , H6 is computed as a function of SR and  $\rho_6$  (table look-up). The value of  $\rho_{6NEW}$  is defined as

$$\rho_{6NEW} = \frac{\rho_{6A} U_{6A}}{\sqrt{2(H_{06A} - H_6)}} \frac{A_{6A}}{A_6}$$

If  $\rho_6$  and  $\rho_{6NEW}$  differ by not more than 0.01 percent, then the iteration is said to have converged. Otherwise, the iterative value of  $\rho_6$  is taken to be the average of  $\rho_6$  and  $\rho_{6NEW}$ , and the process is continued.

- D. The remaining 6 conditions are computed from SR and H6 by interpolation in the tables.
- E. An iteration is performed to find H2. The initial guess on H2 is taken to be<sup>10</sup>

$$H_2 = 0.6H_1 (1 + 0.195X^2)$$

---

<sup>10</sup>Developed empirically from the perfect gas results of Ref. 4, page 60.

where

$$X = 0.32 \frac{U6A}{A6A} + \left( \frac{A6A}{A6} \right)^{0.26} + 1$$

For known H2, U2 can be computed by two relations. The object of this iteration is to find the value of H such that these two values of U, U21 and U22, are equal. The two relations that U must satisfy are

$$1. \quad \rho_2 = \rho_2 (SR, H_2) \text{ (from tape)} \quad (\text{III-1a})$$

$$P_2 = P_2 (SR, H_2) \text{ (from tape)} \quad (\text{III-1b})$$

$$B = \frac{2\rho_2 (H_2 - H_1)}{RT_1} + \rho_2 - P_2 \left( \frac{T_A}{T_1} \right) \quad (\text{III-1c})$$

$$C = -P_2 \rho_2 T_A/T_1 \quad (\text{III-1d})$$

$$\rho_1 = 0.5 (-B + \sqrt{B^2 - 4C}) \quad (\text{III-1e})$$

$$U_{S1} = \left( 2(H_2 - H_1) / \left( 1 - \frac{\rho_1^2}{\rho_2^2} \right) \right)^{1/2} \quad (\text{III-1f})$$

$$P_1 = \rho_1 (T_1/T_A) \quad (\text{III-1g})$$

$$U_{21} = U_{S1} (1 - \rho_1/\rho_2) \quad (\text{III-1h})$$

$$2. \quad U_{22} = U_6 - \frac{a_{ref}}{r_A T_A} \int_{H_6/R}^{H_2/R} \frac{d(H/R)}{a/a_{ref}} \quad (\text{III-2})$$

Equations III-1e through III-1h correspond to Eqs. II-6 through II-9, respectively. Notice that all the quantities necessary to compute U21 and U22 are known for an assumed H2. Newton's method with numerical first derivative is used to find the root of

$$f(H_2) = U_{22} - U_{21} = 0$$

Hence

$$H_{2i+1} = H_{2i} - \frac{f(H_{2i})}{g(H_{2i})}$$

where

$$g(H_{2i}) = \frac{f(H_{2i}) - f(H_{2i-1})}{H_{2i} - H_{2i-1}}$$

As this technique requires two initial guesses, the second initial guess is taken to be 1.1 times the first initial guess. With the

indicated initial guesses and iteration method, the root has converged in every case thus far. The iteration is said to have converged whenever  $H2_{i+1}$  and  $H2_i$  differ by not more than 0.01 percent.

- F. The remaining region 2 conditions are computed in terms of SR and H2.
- G. The characteristic length, L1, is obtained as follows:

$$\Delta t = \frac{US1 - U2}{a2 \quad US1} \quad (\text{III-3a})$$

$$\text{SUM} = \frac{a_a}{r_a \quad T_a} \int_{H6/R}^{H2/R} \frac{d(H/R)}{(a/a_{ref})^2} \quad (\text{III-3b})$$

$$Y = \Delta t \left( \frac{a2}{a6} \right)^{1/2} \exp(0.5 \text{ SUM}/a_{ref}) \quad (\text{III-3c})$$

$$L1 = \frac{1}{(U6 - a6) Y} \quad (\text{III-3d})$$

Input to the program is read from two different tapes.

- A. Tape J1N2 (J1N2 = 10)

This tape contains the gas properties mentioned above. See Subroutine SLOW for the proper format.

- B. Tape J1N1 (J1N1 = 5)

Two read statements are executed by this tape.

1. A title card, Format (72H...)
2. Input data, Format (6E12.0, I2)
  - a. U6A (ft/sec)
  - b. T1 (°K)
  - c. A6AA
  - d. SR
  - e. P6A (atm)
  - f. ALT (ft)
  - g. MORED

All output from the program is on tape JOUT (JOUT = 6).

- A. The title card
- B. SR

C. Inputs and other constants

1. T1
2. H1
3. A6A/A6
4. RHOA
5. PA
6. R
7. SPEED REF

D. Values of the following at the 6-A, 6, and 2 conditions.

1. P
2. U
3. T
4.  $\rho$
5. H
6. A
7. Z

E. Other output

1. L1
2. P1
3. US1
4. MS1
5. DT
6. L8/DTR (for unit L8)

**Subroutine INTRP**

The purpose of this routine is to do an N-point Lagrange interpolation where N-1 is a natural number. The argument list is:

(N, X, Y, XINT, YINT)

N is the number of points

X is the set of independent values<sup>11</sup>

Y is the set of dependent values<sup>11</sup>

XINT is the value of the independent variable at which the interpolation is to take place

YINT is the interpolated value of the dependent variable (the return argument)

---

<sup>11</sup>X and Y should be appropriately dimensioned in the calling routine.



**Subroutine GAUSS**

This subroutine defines constants  $b_i$  and  $x_i$  ( $i = 1, 16$ ) such that

$$\int_p^q f(x) dx$$

can be approximated by

$$\frac{q-p}{2} \sum_{i=1}^{16} b_i f\left(x_i \frac{q-p}{2} + \frac{q+p}{2}\right)$$

The values of  $b_i$  and  $x_i$  were taken from Ref. 17.

The argument list for this subroutine is (b, x). Both b and x should be dimensioned sixteen in the calling program.

**Subroutine SLOW**

The purpose of this subroutine is to do a cross four-point central Lagrange interpolation of data which have been stored on tape as a function of two independent variables. The manner in which the input tape has been created should be equivalent to the following:

```
DO 1 K = 1, N
  1 WRITE (IT) X(K), J, ((Y(K, I, L), I = 1, NV), L = 1, J)
```

where  $4 \leq N$

$2 \leq NV \leq 9$ , a constant defining the number of variables  
exclusive of X and J

$4 \leq J \leq 150$ , a variable defining the number of points for a  
given K

For a given value of K, it is required that  $Y(K, I, L)$  be a strictly monotonic function of L for at least one I. It is also required that  $X(K)$  is a strictly monotonic (increasing or decreasing) function of K.

While X must always be one of the independent variables, the second independent variable and the dependent variable need not be specified until call time. Any Y that is a strictly monotonic function of K can be used as the second independent variable.

The meaning of the variables on the tape associated with this particular problem is as follows:

| Fortran Name | Identification            |
|--------------|---------------------------|
| X(K)         | SR                        |
| J            | variable                  |
| NV           | 9                         |
| Y(K, 1, L)   | T                         |
| Y(K, 2, L)   | $\log_{10} (\rho/\rho_A)$ |
| Y(K, 3, L)   | $\log_{10} (P/P_A)$       |
| Y(K, 4, L)   | $\log_{10} (H/R)$         |
| Y(K, 5, L)   | $\gamma_e$                |
| Y(K, 6, L)   | $a/a_{ref}$               |
| Y(K, 7, L)   | Z                         |
| Y(K, 8, L)   | H/RT                      |
| Y(K, 9, L)   | Z*                        |

The data on this tape were taken from Refs. 6 and 7 primarily; however, certain unpublished extrapolations of the above are also present.

The argument list for the subroutine is

(XX, Z, I1, J1, IT, NV, NERR)

XX is a specified value of SR

Z is a subscripted variable dimensioned appropriately in the calling routine

I1 is a subscript indicating that Z(I1) is the second independent variable

J1 is a subscript indicating that Z(J1) is the dependent variable

IT indicates the channel and unit number on which the input gas tape is mounted

NV indicates the number of variables on the tape corresponding to a value of XX (NV is nine in this case)

NERR will be returned equal to one if and only if the interpolation failed for any reason.

#### Subroutine ATP

This subroutine was not written specifically for this program, and hence has options not used here. Use is made of the subroutine in this program to find temperature and pressure as a function of altitude. The data used by ATP were taken from Ref. 5.

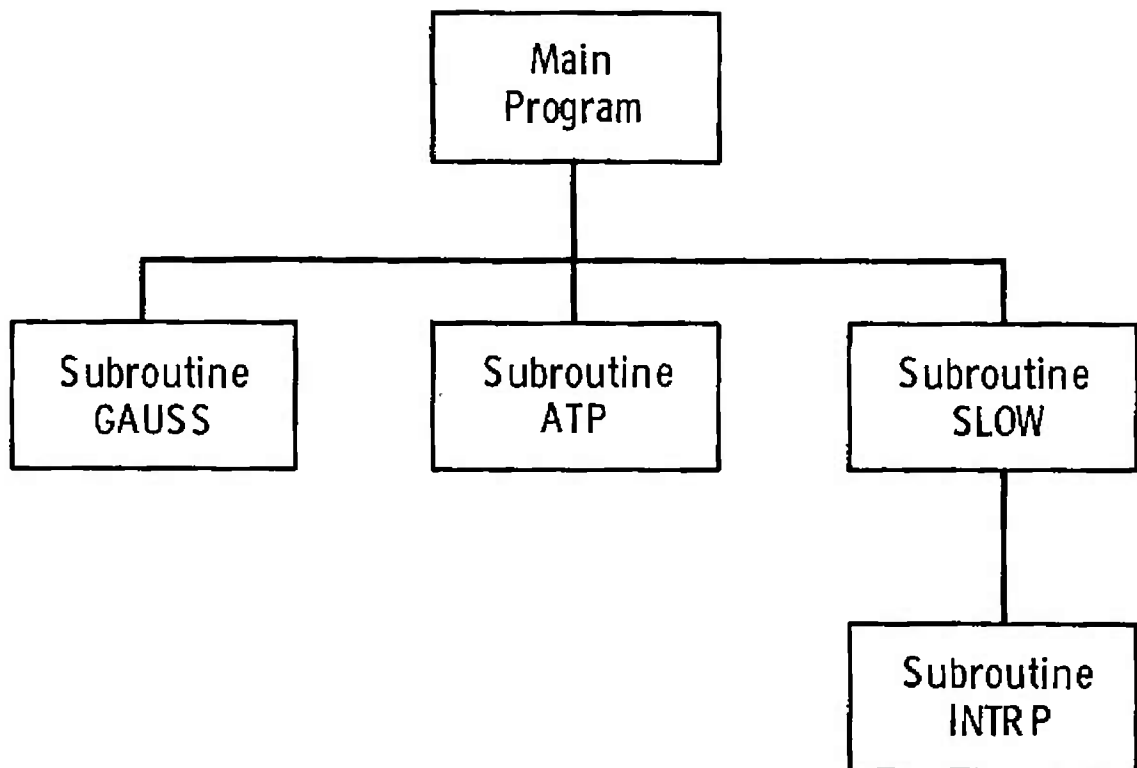


Fig. III-1 Tree Diagram

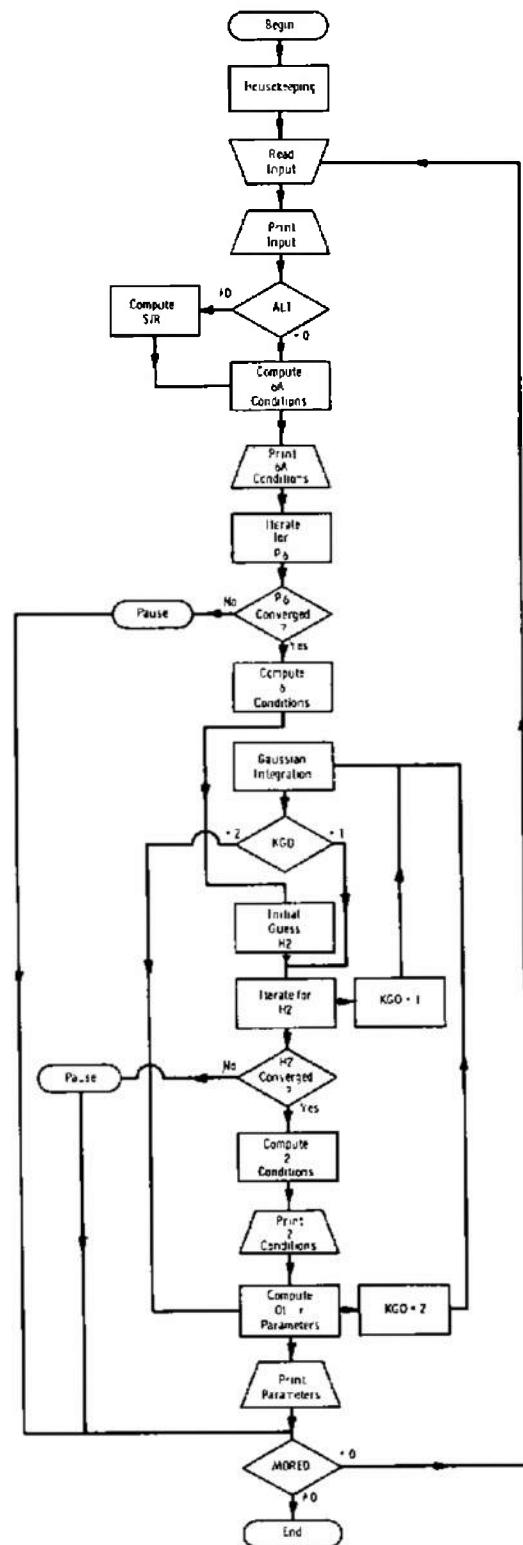


Fig. III-2 Flow Chart of Main Program

**TABLE III-1**  
**CROSS INDEX OF NOMENCLATURE**

| <u>As Used in Appendix III</u> | <u>As Listed in Report Nomenclature</u> |
|--------------------------------|---|
| ALT                            |   |
| SR                             | S/R                                     |
| HO                             | $h_o$                                   |
| HO6                            | $h_{o6}$                                |
| TA                             | $T_a$                                   |
| $\gamma_A$                     | $\gamma_a$                              |
| $a_{ref}$                      | $a_a$                                   |
| R                              | R                                       |
| H1                             | $h_1$                                   |
| T1                             | $T_1$                                   |
| $\rho_1$                       | $\rho_1$                                |
| P1                             | $P_1$                                   |
| US1                            | $U_{S1}$                                |
| H2                             | $h_2$                                   |
| T2                             | $T_2$                                   |
| $\rho_2$                       | $\rho_2$                                |
| P2                             | $P_2$                                   |
| U2                             | $U_2$                                   |
| H6                             | $h_6$                                   |
| T6                             | $T_6$                                   |
| $\rho_6$                       | $\rho_6$                                |
| P6                             | $P_6$                                   |
| U6                             | $U_6$                                   |
| T6A                            | $T_{6a}$                                |
| $\rho_{6A}$                    | $\rho_{6a}$                             |
| U6A                            | $U_{6a}$                                |
| P6A                            | $P_{6a}$                                |
| A6A                            | $a_{6a}$                                |
| a                              | a                                       |

**TABLE III-2**  
**SAMPLE INPUT**

Data Set 1

(A) Card 1 (Title card)

(B) Card 2

1. U6A = 20000.
2. T1 = 296.
3. A6AA = 10.
4. SR = (Not required)
5. P6A = (Not required)
6. ALT = 150,000.
7. MORED = 0 (indicates another set of data follows)

Data Set 2

(A) Card 1 (Title card)

(B) Card 2

1. U6A = 8020.
2. T1 = 296.
3. A6AA = 1.
4. SR = 26.62
5. P6A = .9
6. ALT = (Not required)
7. MORED = 1 (indicates end of job)

TABLE III-3  
SAMPLE OUTPUT

SAMPLE INPUT SHOT ONE UNSTEADY EXPANSION (INPUT ALTITUDE)

S/R=0.30108087E 02

```

INPUT
      T1      H1      A6A/A6      RHOA      PA      R      SPEED REF
0.29600000E 03 0.31997999E 07 0.09999999E 02 0.09999999E 01 0.09999999E 01 0.30917000E 04 0.10873999E 04

6-A CONDITIONS
      P      U      T      RHO      H      A      Z
0.13428957E-02 0.20000000E 05 0.26564383E 03 0.13802770E-02 0.26657203E 07 0.10728073E 04 0.99999994E 00

6-CONDITIONS
      P      U      T      RHO      H      A      Z
0.33525464E-01 0.19784877E 05 0.65598062E 03 0.13952848E-01 0.71796305E 07 0.16668861E 04 0.99999993E 00

2-CONDITIONS
      P      U      T      RHO      H      A      Z
0.22414427E 02 0.73579178E 04 0.29314190E 04 0.20742384E 01 0.39264564E 08 0.33444888E 04 0.10069432E 01

L1=.75029610E-01
P1=.32025871E-00
US1=.85804500E 04
MS1=.75801096E 01
DT=.73562634E-03
LB/DTR=.21504901E 06

```

SAMPLE INPUT SHOT TWO UNSTEADY EXPANSION (INPUT S/R,P/PA)

S/R=0.26620000E 02

```

INPUT
      T1      H1      A6A/A6      RHOA      PA      R      SPEED REF
0.29600000E 03 0.31997999E 07 0.09999999E 01 0.09999999E 01 0.09999999E 01 0.30917000E 04 0.10873999E 04

6-A CONDITIONS
      P      U      T      RHO      H      A      Z
0.90000000E 00 0.80200000E 04 0.62182069E 03 0.39493404E-00 0.67900911E 07 0.16260885E 04 0.10003126E 01

6-CONDITIONS
      P      U      T      RHO      H      A      Z
0.90006664E 00 0.80199999E 04 0.62184337E 03 0.39493404E-00 0.67901570E 07 0.16261183E 04 0.10003123E 01

2-CONDITIONS
      P      U      T      RHO      H      A      Z
0.14919058E 02 0.39505739E 04 0.12788315E 04 0.31728404E 01 0.14795274E 08 0.23004521E 04 0.10043706E 01

L1=.54458258E 00
P1=.67210025E 00
US1=.49104284E 04
MS1=.43379525E 01
DT=.28719155E-03
LB/DTR=.31534562E 05

```

# TABLE III-4

## FORTRAN LISTING

```

SID  LOPER  IBJOB  UNSTEADY EXPANSION
$JOBOPMAP,LOGIC
$IRMAP UNITS
      ENTRY  ,UN10.
,UN10. PZE  UNIT10
UNIT10 FILE  ,B(11,READY,INOUT,BLK=256,LOW,BIN
      FND
$IBFTC MAINP  M94,XR7
C  FORTRAN 4 UNSTEADY EXPANSION PROGRAM
C  SUBROUTINES REQUIRED ARE GAUSS,ATP,SLOW,INTRP
      DIMENSION Z(7),B(16),M(16)
      COMMON IMFT
      CALL GAUSS (B,M)
      RHOA= 1.
      PA= 1.
      PAA= 14.695
      R= 3091.7
      ASPED= 1087.4
      GAMA= 1.4
      TA= 273.15
      CP1= 10810.135
C
C  TAPE ORDER FOR CONSTANT S/R
C1  J  ,NUMBER OF POINTS FOR THIS S/R
C2  T  ,TEMPERATURE
C3  ALOG1(RHO/RHOA)
C4  ALOG(P/PA)
C5  ALOG(H/R)
C6  GAME  ,GAMMA SUB E
C7  A/AA  A/(A SUB A )
C8  Z
C9  H/RT
C10 Z*  Z STAR
C
      5 FORMAT(72H
1
16 FORMAT( 6H0 S/R=E14.8//7H0 INPUT//114H
1  A6A/A6  RHOA  PA  R  M1
2  SPEED REF  /1H 7E16.8 //17H0 6-A CONDITIONS /114H
3  P  U  T  RHO
4  H  A  Z  /1H 7E16.8/)

25 FORMAT (15H0 6-CONDITIONS /114H
1  T  RHO  H  A  U
2  Z  /1H 7E16.8/ 1
64 FORMAT (15H0 2-CONDITIONS /114H
1  T  RHO  H  A  U
2  Z  /1H 7E16.8/ 1
91 FORMAT(5H0 L1=E13.8/5H0 P1=E13.8/6H0 U1=E13.8/6H0 M1=E13.8/5H0 O
1T=E13.8/9H0 L6/D1R=E13.8 )
C  JIN1= INPUT DATA TAPE
JIN1=5
C  JIN2= INPUT GAS PROPERTY TAPE
JIN2=10
C  JOUT= OUTPUT DATA TAPE (THE ONLY ONE)
JOUT=6
NTIME=1
1 CONTINUE
IMFT=0
4 READ (JIN1,5)
WRITE(JOUT,5)
READ (JIN1,2) UKA,T1,A6AA,SR,P6A,ALT,MORED
2 FORMAT(6F12.0,12)
M1= CP)*T1
IF(ALT)200,10,200
200 CALL ATP (1,2,1,2,ALT,T6A,P6A,DIM,DUM)
PAA=P6A/PAA
SR= 23.586 + 3.5*ALOG(T6A/TA)-ALOG(P6A/PA)
10 NFRR=0
Z(7)= ALOG10(P6A/PA)
CALL SLOW (SR,Z,3,1,JIN2,9,NERR )
IF(NFRR-1)11,100,11
11 T6A=Z(1)
CALL SLOW (SR,Z,3,2,JIN2,9,NERR )
12 RHOA= RHOA*10.**Z(2)
CALL SLOW (SR,Z,3,4,JIN2,9,NERR )
13 H6A= R*10.**Z(4)
CALL SLOW (SR,Z,3,6,JIN2,9,NERR )
14 A6A= ASPED*Z(6)
CALL SLOW (SR,Z,3,7,JIN2,9,NERR )
15 Z4A= Z(7)
HOKA= HAA+.5*U6A*U6A
KRITF (JOUT,16)  SR,T1,M1,A6AA,RHOA,PA,R,ASPED,P6A,U6A,T6A,
1PHO6A,H6A,A6A,Z6A

```



TABLE III-4 (Continued)

```

      RHO6=RHO6A*A6AA
      DO 18 I=1,500
      Z(2)= ALOG10(RHO6/RHOA)
      CALL SLOW (SR,Z,4,JIN2,9,NERR )
      IF(NERR-1)17,100,17
17  H6 = R*10,**Z(4)
      TEMP= SQRT(2.*(H06A-H6))
      TFMP= RHO6A*U6A*A6AA/TEMP
      IF(ABS(1-(TFMP-RHO6)/TEMP)*100.-.01)20,20,18
18  RHO6=.5*(RHO6+TEMP)
19  PAUSE 11111
      GO TO 100
20  CALL SLOW (SR,Z,4,1,JIN2,9,NERR )
      U6= RHO6A*U6A*A6AA/RHO6
      IF(NERR-1)21,100,21
21  T6=Z(1)
      CALL SLOW (SR,Z,4,3,JIN2,9,NERR )
22  P6= PA*10,**Z(3)
      CALL SLOW (SR,Z,4,6,JIN2,9,NERR )
23  A6= ASPE0*Z(6)
      CALL SLOW (SR,Z,4,7,JIN2,9,NERR )
24  Z6=Z(7)
      WRITE(JOUT,25) P6,U6,T6,RHO6,H6,A6,Z6
      GO TO 30
C  SUBROUTINE TO COMPUTE INTEGRAL
27  SUM=0.
      I=0
28  I=I+1
      Z(4)= .5*(W(1)*(XU-XL)+XU+XL)
      Z(4)= ALOG10(Z(4))
      CALL SLOW (SR,Z,4,6,JIN2,9,NERR )
      IF (NERR-1)29,100,29
29  SUM= SUM +W(1)/( Z(6)**KGO )
      IF(I-16)28,26,26
26  CONTINUE
      SUM= SUM*(XU-XL)*.5*ASPE0/(GAMA*TA)
      GO TO (35,76)*KGO
C  THE 3-CONDITIONS
30  XX= .32*U6A/A6A + (A6AA)**.26+1.
      H2= H*(1+.195*XX*XX)*.6
      H2=H21

```

```

      DO 50 J=1,500
      Z(4)= ALOG10(H2/R)
      CALL SLOW (SR,Z,4,2,JIN2,9,NERR )
      IF(NERR-1)31,100,31
31  RHO2= RHOA*10.**Z(2)
      CALL SLOW (SR,Z,4,3,JIN2,9,NERR )
32  P2= PA*10.**Z(3)
      BB= (2.*RHO2*(H2-H1)/(R*T1))+RHO2 -P2*TA/T1
      CC= -P2*RHO2*TA/T1
      RHO1= .5*(1-BB+SQRT(BB*BB-4.*CC))
      US1= SQRT(1.+(H2-H1)/(1.-RHO1*RHO2/(RHC2*RHO2)))
      P1=RHO1*T1/TA
      L22= US1*T1.-RHO1/RHO21
      XU= H2/R
      XL= H6/R
      KGO=1
      GO TO 27
35  U21= UA-SUM
      IF(U-1)40,76,40
36  FX1= U21-U22
      H22= 1.1*H21
      GO TO 40
40  FX2= U21-U22
      RATIO=(.-H21/H22)/(1.-FX1/FX2)
      H23= H22*(1.-RATIO)
      IF(ABS((H23-H22)/H23)*100.-.01)60,60,45
45  FX1=FX2
      H21=H22
      H22=H23
49  H2= H22
50  CONTINUE
      PAUSE 22222
      GO TO 100
60  Z(4)= ALOG10(H22/R)
      CALL SLOW (SR,Z,4,1,JIN2,9,NERR )
      IF(NERR-1)61,100,61
61  T2=Z(1)
      CALL SLOW (SR,Z,4,6,JIN2,9,NERR )
62  A2=ASPE0*Z(6)
      CALL SLOW (SR,Z,4,7,JIN2,9,NERR )
63  Z2=Z(7)
      U2=U21

```

TABLE III-4 (Continued)

```

      WRITE(JOUT,64) P2,U2,T2,RHO2,H2,A2,Z2
      CHARACTERISTIC LENGTH
70  DFLT= (US)-U2)/(A2*US))
      Y= DFLT
      XU= H2/P
      XL= H6/R
      KGO=2
      GO TO 27
76  Y=Y*SQRT(A2/A6)*EXP(.5*SUM/ASPED)
      XL1=1./I(U6-A6)*Y)
      XMS1= US)*SQRT(TA/T1)/ASPED
      XLBDT= U*(U6-A6)/A5
      WRITE(JOUT,91)XL1,P1,US1,XMS1,Y,XLBDT
100 CONTINUE
      IF(MORFO)101,4,101
101 RETURN
      END
SUBBTC ATP M94,XR7
C SUBROUTINE ATP = ALTITUDE TEMP AND PRESS (01055/RAMSAY) 01055
      SUBROUTINE ATP(TTEST,JTEST,KTEST,LTEST,A,TA,PA,DA,CSA)
      DIMENSION HB(22),TMB(22),ELM(22),PBI(22),EMB(22)
      DATA(HB(1),I=1,22)/0.,11000.,20000.,32000.,47000.,52000.,61000.,
179000.,88743.,96451.,108129.,117776.,146541.,156071.,165571.,
2184485.,221967.,286476.,376312.,463526.,548230.,630530./
      DATA(TMB(1),I=1,22)/288.15,216.65,216.65,228.65,270.65,270.65,
1252.65,180.65,180.65,210.65,260.65,360.65,760.65,110.65,1210.65,
21350.65,1550.65,1830.65,2160.65,2420.65,2590.65,2760.65/
      DATA(ELM(1),I=1,22)/-.0065,0.,0.1.,.0028,0.,-.002.,-.004,0.,
1.00309023.,.00516636.,.01036592.,.02085668.,.01573977.,.01052632,
2.00740192.,.00533589.,.00434048.,.00367336.,.00298117.,.00200699,
3.00133657.,.00133657/
      DATA(PBI(1),I=1,22)/1513.25,225.32,54.7487,9.58014,1.10905.,.590005,
1.182799.,.6103777,1.6438E-03,3.0075E-04,7.3544E-05,2.5217E-05,
25.0617E-06,3.6943E-06,2.7926E-06,1.6852E-06,6.9604E-07,1.8838E-07,
34.0304E-08,1.0957E-08,3.4502E-09,1.1918E-09/
      DATA(EMB(1),I=1,22)/28.9644,28.9644,28.9644,28.9644,28.9644,
128.9644,28.9644,28.9644,28.9644,28.88,28.56,28.07,26.92,26.66,
226.4,25.65,24.7,22.66,19.94,17.54,16.84,16.17/
      FAC1 = 4.72129
      FAC2 = 0.4788-2
      FAC3 = 1545.31

      FAC5 = 2781.558
      AR = 8314.32
      G = 9.80665
      FMO = 28.9644
      RE = 6.756766.C
      IF(IITEST)217,216,217
216 P1=PA*FAC2
      IF(KTEST-1)218,218,219
218 P1=P1*144.
219 DO 220 I=2,22
      IF(PBI(I)-P1)222,221,220
220 CONTINUE
221 H=HB(I)
      GO TO 223
222 IF(ELM(I)-1)223,236,235
223 H=HR(I-1)+(TMB(I-1)/ELM(I-1))*I*(P1/PBI(I-1))*I*(-AR*ELM(I-1))/
      I*(G*FMO))-1.1
      GO TO 223
226 PFAC=P1/PBI(I-1)
      H=HR(I-1)-(AR*TMB(I-1)*ALOC(PFAC))/G*FMO)
223 Z=IRF(H)/(IRE-H)
225 A=Z/.3048
      GO TO 202
217 Z=A*.3048
200 A=(RE*Z)/(IRE-Z)
202 DO 100 J=2,22
      IF(H-HR(J))101,102,100
100 CONTINUE
101 J=J-1
102 TA=TM(J)+FLMI(J)*(H-HR(J))
      IF(J-22)231,230,230
230 EX=EMB(22)
      GO TO 232
231 EX=EMB(J)-(EMB(J)-EMB(J+1))*(H-HR(J))/(HR(J+1)-HR(J))
232 TA=(EX/EMO)*TA
      CSA=FAC1*SQRT(AR*TA)
      IF(ELM(J))103,104,103
103 P=PBI(J)*(TMB(J)/TMB(J)+ELM(J)*(P-PBI(J)))*I*(G*FMO)/(AR*ELM(J))
      GO TO 105
104 P=PBI(J)*EXP(-(G*ELM(J)+H-HR(J))/AR*TMB(J))
105 GO TO 1105,107,107,JTEST
106 TA=1./P*TA

```

TABLE III-4 (Continued)

```

107 IF(IJTEST)210,21C,215
215 GO TO (108,109),KTEST
108 PA=P*.014503766
      GO TO 210
109 PA=P*2.0885438
210 IF(IJTEST-1)211,211,212
211 FAC35=FAC3
      GO TO 213
212 FAC35=FAC5
213 DA=(FM /FAC35)*(P*2.0885438/TA)
      GO TO (110,214,215),LTEST
315 FAC2=0.
      AR=0.
      CR=0.
      CM=0.
      RE=0.
      FAC1=0.
      FAC3=0.
      FAC5=0.
214 DA=DA/32.174049
110 RETURN
      END
SUBRTC SLOW M94,XRT
      SUBROUTINE SLOW IXX,Z,I1,J1,IT,NV,NERR )
C      TAPE IS WRITTEN WITH LINES OF CONSTANT XX
C      Z(I1) AND XX ARE INDEPENDENT VARIABLES
C      Z(I1) IS THE DEPENDENT VARIABLE
C      AK= +1, IF XX INCREASES MONOTONICALLY ON TAPE
C      AK= -1, IF XX DECREASES MONOTONICALLY ON TAPE
C      IT= TAPE UNIT
C      NV= NO. OF VARIABLES ON TAPE FOR EACH XX (NOT GREATER THAN 9)
C      NO. OF POINTS FOR EACH XX NOT GREATER THAN 150
C      BEGIN EXECUTION
      DIMENSION X(14),Y(14,9,150),Z(9),U(14),V(14),W(14),NP(14)
      COMMON IMFT(20)
      IF(IMFT(1)) 7,1,7
1      BACK SPACE IT
      READ(IT) DUM
      REWIND IT
      DO 2 K=1,3
      READ(IT) X(K),J,((Y(K,I),L),I=1,NV),L=1,J )

2      NP(K)=
      XW=X(2)-X(1)
      AK= ABS(XW)/XW
      DIR1=1.
      IMFT(1)=1
      XXX=XX
      NERR=0
      IM=3
      GO TO 70
7      NERR=0
C      EXCEPT FOR FIRST TIME THROUGH
      IF((XX-X(M1))*(XX-X(M2)))100,100,10
10      TEMP=(XX-XXX)*AK
      DIR2= ABS(TEMP)/TEMP
      GO= DIR1*DIR2
      XXX=XX
      DIR1=DIR2
      IF(DIR2)20,8,50
C
C      NEGATIVE DIRECTION
20      IF(GO)30,8,40
30      BACK SPACE IT
      BACK SPACE IT
      BACK SPACE IT
      GO TO 402
40      IM=IM-1
      IF(IM) 1401,401,402
401      IM=4
402      M1=IM+1
      BACK SPACE IT
      BACK SPACE IT
      IF(IM-4)406,404,403
403      M1=1
404      M2=M1+1
      IF(M2-4)406,406,405
405      M2=1
406      READ(IT) X(M1),J,((Y(M1,I),L),I=1,NV),L=1,J )
      NP(M1)=J
      IF((XX-X(M1))*(XX-X(M2)))100,100,42
42      IF(X(M1)-X(M2))40,43,40
C      ERROR VARIABLE OFF FRONT END OF TAPE
43      CONTINUE

```

TABLE III-4 (Continued)

```

      NERR=1
      GO TO 200
C
C   POSITIVE DIRECTION
50 IF (GO) 60,8,70
60 READ(11) DUM
   READ(11) DUM
   READ(11) DUM
   GO TO 702
70 IM=JM+1
   IF (IM-4) 702,702,701
701 IM=1
702 M1=IM-1
   IF (M1) 703,703,704
703 M1=4
704 M2=M1-1
   IF (M2) 705,705,706
705 M2=4
706 READ(11) X(IM),J,((Y(IM,I),I=1,NV),L=1,J)
   NPI(M)=J
   IF ((XX-X(M1))*(XX-X(M2))) 100,100,70
C
C   TAPE SEARCH COMPLETE , DO CROSS FOUR POINT
100 DO 150 K=1,4
   NPK=NPK+1
   DO 115 I=1,NPK
   IF (Y(K,I)-Z(I1))*Y(K,I+1)-Z(I1)) 125,125,115
115 CONTINUE
   NERR=1
   GO TO 200
125 IF (I-1) 127,126,127
126 J=0
   GO TO 137
127 IF (I-NPK) 136,135,136
135 J=NPK-3
   GO TO 137
136 J=I-2
137 DO 140 L=1,4
   MX=L+J
   U(L)= Y(K,I,MX)
140 V(L)= Y(K,J,MX)

150 CALL INTRP (4,U,V,Z(I1),W(K))
   CALL INTRP (4,X,W,XX,Z(I1))
175 RETURN
8 CONTINUE
200 NFRP=1
   IF (IMET(2)) 215,201,215
201 IMET(2)=1
   DO 210 JM=1,4
   WRITE(6,202) X(JM)
202 FORMAT(1H'E16.8)
203 FORMAT(1H' 7E16.8)
   NXXXX=NPK(JM)
210 WRITE(6,203) ((Y(JM,I),I=1,NV),L=1,NXXXX)
215 RETURN
   END
SIFTC GAUSS 494,XR7
SUBROUTINE GAUSS (B,X)
C   GAUSS CONTAINS EIGHT PLACE,SIXTEEN POINT INTEGRATION CONSTANTS
   DIMENSION B(16),X(16)
   X(1)=.095012510
   X(2)=.28160355
   X(3)=.45801678
   X(4)=.61787624
   X(5)=.75540441
   X(6)=.85563120
   X(7)=.94457502
   X(8)=.98940093
   B(1)=.18945061
   B(2)=.19260342
   B(3)=.16915652
   B(4)=.14959599
   B(5)=.12462897
   B(6)=.095158512
   B(7)=.062253524
   B(8)=.027152450
   DO 12345 I=9,16
   J=17-I
   X(I)=X(J)
   B(I)=B(J)
12345 RETURN
   END
<1BFTC INTRP 494,XR7

```

TABLE III-4 (Concluded)

```

C      THIS IS AN N-POINT LAGRANGE INTERPOLATION SUBROUTINE (N.GT.2)
SUBROUTINE INTRP (N,X,Y,XINT,YINT)
  DIMENSION X(1),Y(1)
  YINT=0
  DO 202 I=1,N
    SUMN=1
    SUMD=1
    DO 201 J=1,N
      IF (J-I) 200,201,200
    200 SUMN=SUMN*(XINT-X(J))
      SUMD=SUMD*(X(1)-X(J))
    201 CONTINUE
    202 YINT=YINT+Y(I)*SUMN/SUMD
  RETURN
END

```

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